Maximal endurance time at $\dot{V}O_{2max}$

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ABSTRACT

MORTON, R. H., and V. BILLAT. Maximal endurance time at $\dot{V}O_{2max}$. *Med. Sci. Sports Exerc.*, Vol. 32, No. 8, pp. 1496–1504, 2000. **Introduction:** There has been significant recent interest in the minimal running velocity which elicits $\dot{V}O_{2max}$. There also exists a maximal velocity, beyond which the subject becomes exhausted before $\dot{V}O_{2max}$ is reached. Between these limits, there must be some velocity that permits maximum endurance at $\dot{V}O_{2max}$, and this parameter has also been of recent interest. This study was undertaken to model the system and investigate these parameters. **Methods:** We model the bioenergetic process based on a two-component (aerobic and anaerobic) energy system, a two-component (fast and slow) oxygen uptake system, and a linear control system for maximal attainable velocity resulting from declining anaerobic reserves as exercise proceeds. Ten male subjects each undertook four trials in random order, running until exhaustion at velocities corresponding to 90, 100, 120, and 140% of the minimum velocity estimated as being required to elicit their individual $\dot{V}O_{2max}$. **Results:** The model development produces a skewed curve for endurance time at $\dot{V}O_{2max}$, with a single maximum. This curve has been successfully fitted to endurance data collected from all 10 subjects ($R^2 = 0.821$, $P < 0.001$). For this group of subjects, the maximal endurance time at $\dot{V}O_{2max}$ can be achieved running at a pace corresponding to 88% of the minimal velocity, which elicits $\dot{V}O_{2max}$ as measured in an incremental running test. Average maximal endurance at $\dot{V}O_{2max}$ is predicted to be 603 s in a total endurance time of 1024 s at this velocity. **Conclusion:** Endurance time at $\dot{V}O_{2max}$ can be realistically modeled by a curve, which permits estimation of several parameters of interest; such as the minimal running velocity sufficient to elicit $\dot{V}O_{2max}$, and that velocity for which endurance at $\dot{V}O_{2max}$ is the longest. **Key Words:** ANAEROBIC CAPACITY, CRITICAL VELOCITY, MINIMAL $\dot{V}O_{2max}$ VELOCITY, MODELING, OXYGEN UPTAKE, SLOW COMPONENT

The relationship between power output and endurance time is a fertile area for the study of human bioenergetics and work performance. For two recent reviews, consult Billat et al. (2) and Morton and Hodgson (25). With very few exceptions, this research has focused on endurance at constant powers, where the critical power (CP) concept (16) has been by far the most commonly adopted model. It has been widely studied and adapted for swimming, running, rowing, cycling, kayaking, and wheelchair exercise. Nevertheless, it is not without its critics (31).

For some of these exercise modalities, power output can be measured directly on an ergometer. However, for running, swimming, and wheelchair exercise, velocity and distance take the place of power and work, respectively, with corresponding changes to the units of measurement of the parameters of the model. It would be useful if a single forcing variable could be found, one which is independent of exercise modality and which could be used in a totally general setting. Oxygen uptake may be one such candidate.

Oxygen uptake, however, is not a simple function of power output or velocity, for it is a function of time as well. Even steady-state oxygen uptake is not a linear function of power output beyond a certain level. The slow component of oxygen uptake and increasing oxygen cost of exercise at higher powers complicates the issue (14). The slow component has, however, been successfully modeled, both theoretically (22) and empirically (1); and the energy cost of running can safely be assumed constant (or very nearly so) provided the power or velocity range is narrow. These models are not mathematically simple. Perhaps then these difficulties can be largely overcome by considering endurance at a fixed value of oxygen uptake, say at its maximum ($\dot{V}O_{2max}$).

The power range that will bring on exhaustion in a finite time can be divided into three domains. Power output may be high, that is higher than CP but insufficient to elicit $\dot{V}O_{2max}$. It may be very high or maximal, sufficient to drive $\dot{V}O_2$ to its maximum before exhaustion. Or it may be extreme, such that the subject becomes exhausted before sufficient time has elapsed for $\dot{V}O_2$ to reach its maximum. Indeed, the minimum power or velocity just sufficient to elicit $\dot{V}O_{2max}$ before exhaustion in a subject, and endurance time at $\dot{V}O_{2max}$, are two phenomena of current interest to exercise physiologists, sports coaches, and athletes in training.

The relationship between power output and total endurance has been modeled over the whole power range above CP as referred, but modeling endurance at $\dot{V}O_{2max}$ is restricted to the narrower mid-range. Given what is already known about the human exercise response, it should be possible to model this latter relationship, perhaps in a similar way to modeling endurance at constant power. To our

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knowledge, this has never been accomplished, and it is the purpose of this paper to take a tentative step forward in this modeling process.

THE SIGNIFICANCE OF THE MINIMAL VELOCITY AND THE RANGE OF VELOCITIES THAT ELICIT VO_{2MAX}

Over 75 yr ago, Hill and Lupton (15) recognized that $\dot{V}O_2$ reached its maximum "... for speeds beyond about 256 meters per minute." At higher speeds, the oxygen requirement is higher but cannot be satisfied. This minimal velocity Volkov et al. (32) called "critical speed," not to be confused with the critical power concept mentioned above. Volkov et al. investigated endurance at this critical speed as a means of assessing maximal aerobic capacity. It was Daniels et al. (10) who introduced the term "velocity at $\dot{V}O_{2max}$" and the abbreviation "$v\dot{V}O_{2max}$," reporting it as a useful variable that combined $\dot{V}O_{2max}$ and running economy into a single factor that showed good potential for identifying aerobic differences between various runners or categories of runners.

Time to exhaustion at $v\dot{V}O_{2\ max}$ is reproducible in any one subject, but there is great variability between subjects even when the coefficient of variation for $\dot{V}O_{2max}$ is low (4). It seems that the lactate threshold, which is correlated with time to exhaustion at $v\dot{V}O_{2\ max}$, can explain these differences, though the role of the anaerobic contribution is not negligible (13). An inverse relationship has been found between $v\dot{V}O_{2\ max}$ and $\dot{V}O_{2max}$ itself, and a positive relationship between $v\dot{V}O_{2\ max}$ and the velocity at the lactate threshold expressed as a fraction of $v\dot{V}O_{2\ max}$ (5). These results are similar for different sports (6). Several other studies confirm the value of $v\dot{V}O_{2\ max}$ and time to exhaustion at this velocity when analyzing the performances of runners over distances from 1500 m to the marathon (6,11,21,26).

However, as indicated in the introduction, at velocities above that corresponding to the time-velocity asymptote (critical velocity), a slow component of $\dot{V}O_2$ kinetics becomes manifest maybe as much as 60 or more seconds after exercise onset (14,34).

One obvious consequence of the slow component response is that it creates a range of velocities, all of which elicit $\dot{V}O_{2max}$ provided exercise is continued to exhaustion. Thus the so-called "velocity associated with $\dot{V}O_{2max}$" defined as the minimal velocity that elicits $\dot{V}O_{2max}$ for incremental exercise (3) would not be the sole velocity that elicits $\dot{V}O_{2max.}$ Indeed, $\dot{V}O_{2max}$ can be elicited during constant power exercise over a range of intensities that may be higher or lower than the minimal value for which it occurs during incremental exercise (33). However, in that range of exercise intensities, there must be one value which would allow the longest endurance at $\dot{V}O_{2max}$, which by definition could be the asymptote of the time at $\dot{V}O_{2max}$–velocity relationship. Because the time-velocity relationship is strong for high-intensity exercise that leads to exhaustion within 1–30 min (14), we could use four all-out runs, say at 90, 100, 120, and 140% of $v\dot{V}O_{2\ max}$ to determine the asymptote.

In a recent study (7), where the purpose was to determine the velocity which permits the longest endurance at $v\dot{V}O_{2max}$, it was calculated by the slope of the distance/time regression line obtained from four such runs. This study reported that this velocity was not significantly different from that determined in a 3-min stage incremental protocol in six physical education student subjects. This velocity was higher than the critical velocity computed using total distance and time. This could be explained by the observation that five of the six subjects did not reach $\dot{V}O_{2max}$ at 90% of $v\dot{V}O_{2max}$, leading to a very low average for endurance times at $\dot{V}O_{2max}$ in the 90% runs. That is, times and distances at $\dot{V}O_{2max}$ in these four runs used to determine the minimal velocity which elicits $\dot{V}O_{2max}$ (by allowing the slow component to reach $\dot{V}O_{2max}$) were shorter than total times and distances to exhaustion at these velocities. Times at $\dot{V}O_{2max}$ were also much lower for the 120 and 140% runs (73 s and 18 s, respectively). Hence, the critical velocity calculated only with times and distances at $\dot{V}O_{2max}$ was not significantly different from the velocity that elicits $\dot{V}O_{2max}$ determined in the incremental protocol.

This result was in accordance with Monod and Scherrer (20), who originally considered that the critical velocity corresponded to a power output at $\dot{V}O_{2max}$. In the same way, Ettema (12) considered that the critical velocity was close to the velocity corresponding to the maximal oxygen uptake in world class runners. In one sense they were right, accepting the fact that to be true only the time spent at $\dot{V}O_{2max}$ for each power output must be taken into account and that it is impossible to sustain this velocity indefinitely.

Indeed, the main criticism of the two-parameter critical power model is the assumption that the critical (aerobic) power, i.e., $\dot{V}O_{2max}$, is attained at the very onset of work. This flaw is avoided when the distance is plotted versus the time, only after $\dot{V}O_{2max}$ is actually attained.

The interest in the velocity which permits $\dot{V}O_{2max}$ to be sustained the longest time, is the belief that it could be a good intensity to train at for middle and long distances (3,8). However, further research is required to determine and understand this velocity, using velocities closer to $v\dot{V}O_{2max}$ (95, 100, 105, and 110% of $v\dot{V}O_{2max}$, for example). Indeed, a slow component of $\dot{V}O_2$ could appear at 95% of $v\dot{V}O_{2max}$ and would induce a decrease of the critical velocity calculated with the longest time at $\dot{V}O_{2max}$ only. Therefore, the critical velocity could be the velocity which elicits the longest time at $\dot{V}O_{2max}$. This definition could allow clarification of the critical velocity concept and delineation of the range of velocities for which a $\dot{V}O_2$ slow component appears, helping in further studies to elucidate its underlying mechanisms.

METHODS

Model background. Quite apart from any philosophical arguments concerning the CP concept itself, there are several practical ones to consider. Several of these are discussed by Vandewalle et al. (31). It is of value in setting the scene for modeling endurance at $\dot{V}O_{2max}$ to recall the more relevant of these.

It has been clearly shown that when subjects have their endurance at CP (as estimated from the model) tested, they are seldom able to endure for 1 h, often much less (17–19,27). Certainly this falls well short of the "infinitely" long endurance predicted by the model. As a consequence, the work-time relationship is not linear but curved downward, and the resulting parameter estimates depend on the selection of powers for the experimental determination of CP (9).

At the other extreme, the CP model predicts an infinitely high power as endurance time shrinks to zero. Clearly this cannot be so, as the concept of maximal anaerobic power is well established (30). Some finite maximal "instantaneous" power must exist, beyond which no work can be performed and endurance time is zero.

As a consequence, one can deduce that some self-preservation control system must be in operation, and that the assumption of the CP model that at exhaustion all of the anaerobic capacity is completely consumed, is erroneous. Indeed, Saltin and Karlsson (29) have clearly demonstrated the existence of significant anaerobic reserves at exhaustion at various power outputs.

All of the above difficulties have already been overcome by the adoption of a linear control system for power output based on the extent to which the anaerobic capacity has been consumed. The resulting three-parameter critical power model is fully discussed by Morton (24). Nevertheless, two further practical difficulties remain.

First, the adjustment of oxygen delivery to the working muscles as required by the exercise, is not instantaneous as assumed by the CP model. In fact it may take 2 or 3 min to reach the required level. Wilkie (35) has recognized this problem, though his formulation has other difficulties in common with the CP model. Peronnet and Thibault (28) also recognized this problem, as well as a declining ability to sustain power output at high fractions of $\dot{V}O_{2max}$. Their model, however, does contain several arbitrary components, and is not simple. Second, and perhaps most problematic of all, the anaerobic reserves are not comprised of a single component, but at least two, accessible through different metabolic pathways. The three-component hydraulic model of human bioenergetics proposed by Morton (22) has addressed both these problems. The former is straightforward to overcome, but the latter adds significant complexity to the modeling. Nevertheless, this model has been extended to investigate maximum power and endurance by the introduction of a control system (23).

The approach taken in this paper is therefore to construct a model (see Appendix), building on the previous work of several authors, which represents an energy demand and supply system with the following properties:

- It is based on the two component aerobic and anaerobic critical power concept adapted to running.
- It incorporates a linear control system for power output, dependent on the amount of anaerobic reserve consumed.
- Aerobic energy supply adjusts as a response with single fast exponential kinetics up to the level of critical power.
- It incorporates the slow component (a second exponential), which drives $\dot{V}O_2$ beyond the equivalent of CP toward $\dot{V}O_{2max}$.

In so doing, endurance time both in total and at $\dot{V}O_{2max}$ can be modeled as a system with six parameters. They are the anaerobic distance capacity (α); critical velocity (CV); maximal "instantaneous" velocity (V_{max}); the minimal velocity sufficient to elicit $\dot{V}O_{2max}$ (V_{vm}); and two kinetic rate parameters for the fast (r_1) and slow (r_2) components of oxygen uptake.

Important assumptions. A number of assumptions are inherent in the preceding discussion and in the basis on which the model is constructed as described above. Two of these deserve particular mention.

We assume that at the time $\dot{V}O_2$ reaches the equivalent of CV, the primary exponential component is just complete, or very nearly complete, at which point the slow component of $\dot{V}O_2$ begins. This assumption allows $\dot{V}O_2$ above the equivalent of CV to be treated as a single slow rate exponential process with delay, thus simplifying the mathematics significantly. For the theoretical model of Morton (22) and its empirical verification by Barstow and Molé (1), this seems to be fairly reasonable, as the time of commencement of the slow component occurs part way into the exercise. However, for extremely high exercise levels, $\dot{V}O_2$ may reach the equivalent of CV quite quickly, say within 30 s or less. In such cases, the degree of simplification our model assumes becomes more important, though this is irrelevant if exhaustion occurs before $\dot{V}O_{2max}$ is reached.

We also assume that once $\dot{V}O_2$ reaches the equivalent of CV, the contribution of the aerobic power source to the requirement of exercise stabilizes at this level. This derives directly from the usual interpretation of the critical power concept. This is despite the fact that the slow component may drive $\dot{V}O_2$ significantly beyond the equivalent of CV. In other words, the contribution which the slow component of $\dot{V}O_2$ makes does not enter into energy supply/demand considerations. Indeed Barstow and Molé (1) conjecture whether the slow component ". . . represents some energy consuming function that is ancillary to, or even completely separate from, contraction." We assume that it does. If it does not, then the critical power concept needs major reinterpretation.

The full model development, including a glossary of terms is detailed in the Appendix.

Subjects. Ten physically active male subjects (mean ± SD, age 26.4 ± 4.7 yr, weight 79.1 ± 4.5kg, $\dot{V}O_{2max}$ 59.3 ± 5.0 mL·kg^{-1}·min^{-1}) volunteered for this study. Each subject was familiar with the experimental procedures before the study, and all gave their written informed consent to participate in accordance with the French National Committee for Clinical Research.

Laboratory procedures. Subjects reported to the laboratory fasted and hydrated on five occasions. On the first occasion, $\dot{V}O_{2max}$ and $v\dot{V}O_{2\ max}$ were measured using an incremental test protocol on a treadmill (Gymrol

TABLE 1. Individual total times and distances run during the all-out runs at 90, 100, 120, and 140% of $v\dot{V}O_{2max}$, and specific times and distances run at $\dot{V}O_{2max}$.

Subject No.	$\dot{V}O_{2max}$ Incremental Test ($mL \cdot kg^{-1} \cdot min^{-1}$)	Absolute Velocity ($m \cdot s^{-1}$)	Relative Velocity % $v\dot{V}O_{2max}$	Total Time Limit (s)	Time Limit at $\dot{V}O_{2max}$ (s)	Total Distance Limit (m)	Distance Limit at $\dot{V}O_{2max}$ (m)
1	63	4.00	90	975	0	3900	0
		4.44	100	450	240	2000	1067
		5.33	120	150	90	480	480
		6.22	140	75	30	187	187
2	55	4.00	90	1200	0	4800	0
		4.44	100	405	210	1800	933
		5.33	120	118	75	629	400
		6.22	140	65	15	404	93
3	59	4.25	90	800	600	3400	2550
		4.72	100	388	300	1832	1417
		5.19	120	138	75	717	390
		6.61	140	89	60	588	397
4	66	4.44	90	840	0	3733	0
		5.00	100	225	100	1125	600
		6.00	120	75	25	450	180
		7.00	140	45	0	315	0
5	55	4.25	90	805	525	3421	2231
		4.72	100	337	225	1591	1062
		5.19	120	142	105	737	545
		6.61	140	81	60	535	396
6	62	4.44	90	495	95	2200	422
		5.00	100	210	135	1050	675
		6.00	120	90	45	540	270
		7.00	140	60	0	420	0
7	62	4.00	90	770	0	3080	0
		4.44	100	360	150	1600	667
		5.33	120	135	90	720	480
		6.22	140	75	0	466	0
8	60	4.12	90	1125	870	4640	3588
		4.58	100	524	420	2401	1925
		5.50	120	110	45	605	247
		6.42	140	70	30	449	193
9	49	4.00	90	420	0	1680	0
		4.44	100	270	150	1200	667
		5.33	120	135	90	720	480
		6.22	140	90	45	560	0
10	62	4.44	90	885	0	3933	0
		5.00	100	480	345	2400	1725
		6.00	120	150	105	900	630
		7.00	140	90	30	630	210

1800). At the start, speed was set at 12 $km \cdot h^{-1}$ (0% slope) and was increased by 2 $km \cdot h^{-1}$ every 3 min up to 80% of their running speed in a 1.5-km race and by 1 $km \cdot h^{-1}$ every 3 min, thereafter, until exhaustion. The criteria used for $\dot{V}O_{2max}$ were a plateau in $\dot{V}O_2$ despite increasing running speed, a respiratory exchange ratio above 1.1, and HR above 90% of the age-predicted maximum. $v\dot{V}O_{2max}$ was the lowest running velocity that elicited $\dot{V}O_{2max}$ as defined. The four endurance tests at 90, 100, 120, and 140% of $v\dot{V}O_{2\ max}$ were subsequently performed in random order for each subject at sessions separated by at least 1 wk. At each, after a 5-min warm-up at 60% of their $v\dot{V}O_{2\ max}$, speed was quickly increased (over less than 20 s) up to the required velocity. Subjects were verbally encouraged to run to exhaustion. The total endurance time and distance covered together with the endurance time, and distance covered at $\dot{V}O_{2max}$ were recorded at each session for each subject.

Curve fitting. The equation modeling endurance time at $\dot{V}O_{2max}$ (see Appendix) was fitted to the pooled subject data using the least squares procedure incorporated in SigmaPlot software (Jandel Scientific, San Rafael, CA).

RESULTS AND DISCUSSION

Table 1 lists individual results from all 10 subjects.

For six subjects, 90% of their $v\dot{V}O_{2max}$ as measured on the incremental test was insufficient to elicit their individual $\dot{V}O_{2max}$. For two of these, and one other, 140% of their $v\dot{V}O_{2max}$ brought on exhaustion before their $\dot{V}O_{2max}$ could be attained. Those three subjects who were able to reach their $\dot{V}O_{2max}$ on all four tests, produced data which appeared skew, with endurance at 90% much longer than at any other percentage.

Equation 6 for endurance time at $\dot{V}O_{2max}$ (see Appendix) contains six parameters, but it can be parameterized more simply as

$$t_a = a/(V - b) + c\,ln(1 - d/(V - b)) - e$$

which contains only five independently estimable parameters (the original formulation contains one redundant parameter). However, because there are at most four nonzero data points for any one subject, this equation cannot be fitted for each subject, and so the data must be pooled over subjects. Furthermore, we note that equation 4 for total endurance time (see Appendix) contains four parameters

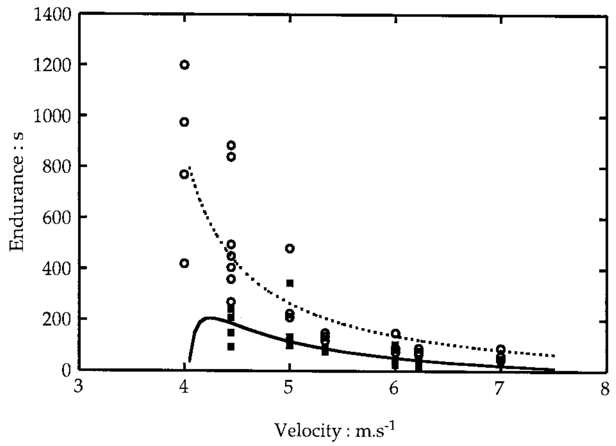

Figure 1—Endurance data and jointly fitted curves for seven subjects. This figure shows individual endurance times (both total and at $\dot{V}O_{2max}$) versus running velocities for subjects 1, 2, 4, 6, 7, 9, and 10, together with fitted model curves for this group. *Open symbols* and *dotted fit* are for total endurance times, *closed symbols* and *solid fit* are for endurance times at $\dot{V}O_{2max}$. Goodness of fit and estimated parameters are as given in the text.

(total endurance time provides no information on V_{vm} or on the rate constant for the $\dot{V}O_2$ slow component).

Thus, to overcome the redundancy and provide for more efficient estimation of the four parameters common to equations 4 and 6, these two equations are fitted jointly to both sets of endurance data pooled over subjects. This was done using the nonlinear least squares procedure in SigmaPlot (Jandel Scientific, San Rafael, CA). In the first instance, the skewed data was omitted, and 47 data points from seven subjects were used.

Although the fitted parameters are a little sensitive to the starting values in the iterative least squares routine, R^2 values in the range 0.736–0.752 can be obtained ($P < 0.01$ in all cases). This is acceptable, but not particularly good, because the standard error of the estimate ranged between 142–176 s. Nevertheless, the fitted parameters all had ranges which would be considered normal in a population of subjects as reflected by the 10 in this study.

Typically, the anaerobic distance capacity (α) is estimated in the range 449–548m, critical velocity (CV) in the range 3.45–3.74 $m{\cdot}s^{-1}$, rate constant for primary $\dot{V}O_2$ kinetics (r_1) in the range 0.0254–0.0432 s^{-1} (a time constant in the range 23.2–39.4 s), rate constant for the $\dot{V}O_2$ slow component (r_2) in the range 0.00324–0.00482 s^{-1} (time constant 207.5–308.6 s), maximal "instantaneous" velocity (V_{max}) in the range 9.1–12.3 $m{\cdot}s^{-1}$, and minimal velocity to elicit $\dot{V}O_{2max}$ (V_{vm}) in the range 3.9–4.0 $m{\cdot}s^{-1}$.

As an illustration of the fit, Figure 1 above shows the raw data and jointly fitted curves for these seven subjects. In this fit, $R^2 = 0.734$, $\alpha = 527$m, CV = 3.5 $m{\cdot}s^{-1}$, $r_1 = 0.0423$, $r_2 = 0.00338$, $V_{vm} = 4.0$, and $V_{max} = 12.19$ $m{\cdot}s^{-1}$.

From this fit, we note that time at $\dot{V}O_{2max}$ is zero for V = 4.04 $m{\cdot}s^{-1}$, which is just above V_{vm} of 4.0 $m{\cdot}s^{-1}$, and also for V = 8.25 $m{\cdot}s^{-1}$, which is somewhat below V_{max} of 12.19 $m{\cdot}s^{-1}$.

In an effort to improve the fit and avoid some of the above difficulties, we can utilize the relative velocities expressed as % of $v\dot{V}O_{2max}$ (measured during the incremental test) as the independent variable. This, however, introduces a differential scaling, because V_{vm} is not the same for all subjects. Also, it alters the units of measurement of several of the parameters of the fitted equations; velocities become dimensionless (expressed as percentages) and the anaerobic distance capacity becomes measured in hundredths of a second.

In so doing, the fit is better, $R^2 = 0.839$ ($P < 0.0001$) with a standard error of the estimate of 117.3 s, and the fitted values are insensitive to any reasonable starting values. α is estimated as 7687.1 (76.87 s), which corresponds to an anaerobic distance capacity of about 360m; CV is estimated as 81.5% of $v\dot{V}O_{2max}$ (incremental) or about 3.82 $m{\cdot}s^{-1}$; V_{vm} is estimated as 89.5% of $v\dot{V}O_{2max}$ (incremental) or about 4.20 $m{\cdot}s^{-1}$; V_{max} is estimated as 208% $v\dot{V}O_{2max}$ (incremental), which corresponds to a velocity of about 9.75 $m{\cdot}s^{-1}$, r_1 is estimated as 0.167 s^{-1}, which corresponds to a rather rapid time constant of 6 s; and r_2 is estimated as 0.00419 or a time constant of 239 s.

Of most interest here perhaps is the observation that the estimate of $v\dot{V}O_{2max}$ (incremental) is larger by about 10%, than the fitted V_{vm}. Because incremental velocity steps are in 1 or 2 $km{\cdot}h^{-1}$, this is perhaps not surprising. Indeed, from Table 1 the average observed $v\dot{V}O_{2max}$ for these seven subjects is 4.68 $m{\cdot}s^{-1}$, and the earlier fitted value of 4.0 $m{\cdot}s^{-1}$ is a slightly lesser 85.4% of this.

Data from three subjects were omitted, because of uncommonly long endurance times at 90% of $v\dot{V}O_{2max}$. If these were to be plotted on Figure 1, the skewness would be obvious. However, in view of the above discussion, the effect of their inclusion should be investigated. This was done in a joint fit of equations 4 and 6 now utilizing all 71 nonzero data points in Table 1, for both velocity and % $v\dot{V}O_{2max}$ as independent variables. Fitting velocity against endurance times provides a slightly better fit than before, $R^2 = 0.768$ ($P < 0.001$) with a standard error of estimate of 145.3 s. The fitted parameters are $\alpha = 914$m, CV = 3.35 $m{\cdot}s^{-1}$, $r_1 = 0.0128$ s^{-1}, $V_{vm} = 3.36$ $m{\cdot}s^{-1}$, $r_2 = 0.000106$, and $V_{max} = 7.98$ $m{\cdot}s^{-1}$.

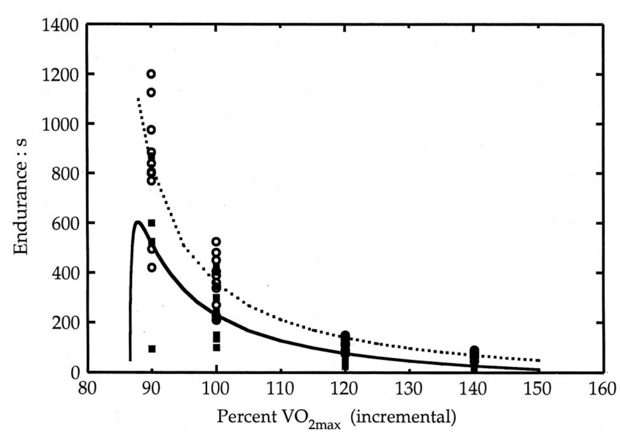

Figure 2—Endurance data and jointly fitted curves for 10 subjects. This figure shows individual endurance times (both total and at $\dot{V}O_{2max}$) versus % $v\dot{V}O_{2max}$ for all 10 subjects; together with fitted model curves for the whole group. *Open symbols* and *dotted fit* are for total endurance times, *closed symbols* and *solid fit* are for endurance times at $\dot{V}O_{2max}$. Goodness of fit and estimated parameters are as given in the text.

Fitting % v$\dot{V}O_{2max}$ against endurance times yields a still better fit, $R^2 = 0.821$ ($P < 0.001$) with a standard error of estimate of 127.6 s. the fitted parameters are $\alpha = 89.01$ s corresponding to 416m, CV = 81.4% v$\dot{V}O_{2max}$ corresponding to 3.81 m·s^{-1}, $r_1 = 0.0535$ s^{-1} corresponding to a time constant of 18.7 s, $V_{vm} = 86.5\%$ v$\dot{V}O_{2max}$ corresponding to 4.05 m·s^{-1}, $r_2 = 0.0034$ corresponding to a time constant of 292.7 s, and $V_{max} = 203.7\%$ of v$\dot{V}O_{2max}$ corresponding to 9.53 m·s^{-1}. These, as a collection of values, are more coherent than the former. These fitted equations are plotted together with the full data set in Figure 2. The increased skewness of the curve for endurance time at $\dot{V}O_{2max}$ is immediately apparent by comparison with Figure 1. The maximal endurance time at $\dot{V}O_{2max}$ is predicted as 603 s for a running velocity of 4.11 m·s^{-1} being 87.9% of v$\dot{V}O_{2max}$ (incremental).

CONCLUSIONS

We have agreed that there must exist some running velocity in the range of velocities that elicit $\dot{V}O_{2max}$, which permits maximal endurance at $\dot{V}O_{2max}$. Indeed, we have shown that whereas total endurance time plotted against velocity displays a hyperbolic shape, endurance time at $\dot{V}O_{2max}$ plotted against velocity displays a maximum. The bioenergetic process that produces such joint data has been modeled, producing a skewed curve for endurance at $\dot{V}O_{2max}$, which does have a single maximum. This model has been successfully fitted to endurance data, both in total and at $\dot{V}O_{2max}$, obtained from a group of 10 subjects. We find that the fitted minimal velocity to elicit $\dot{V}O_{2max}$ is some 10–13% below that estimated from an incremental test and that maximal endurance at $\dot{V}O_{2max}$ is achieved running at not much above this fitted value.

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APPENDIX

Glossary

- AC — Anaerobic capacity, usually expressed in joules, but here expressed as its distance equivalent, m.
- α — The value of AC when the subject is fully rested and nourished, m.
- CP — Critical power, W.
- CV — Critical velocity, m·s^{-1}.
- e — The dimensionless exponential constant.
- *ln* — The natural logarithm to the base *e*.
- r_1 — The rate constant for the primary component of oxygen uptake kinetics, s^{-1}.
- r_2 — The rate constant for the slow component of oxygen uptake kinetics, s^{-1}.
- t — The general time variable, s.
- t_a — The endurance time at $\dot{V}O_{2max}$, s.
- t_c — The time taken for the primary component of oxygen uptake to reach the equivalent of CV, s.
- t* — Total endurance time at constant velocity, s.
- t_{vm} — The time taken for oxygen uptake to reach $\dot{V}O_{2max}$, s.
- V — The constant velocity required of any given exercise bout, m·s^{-1}.
- V_m — The maximal velocity that could be attained when the anaerobic capacity is less than α, m·s^{-1}.
- V_{max} — The maximal velocity achievable by the subject when fully rested and nourished, m.
- $\dot{V}O_2$ — Oxygen uptake above rest, usually expressed in L·min^{-1} or mL·kg^{-1}·min^{-1}, but here expressed as its velocity equivalent, m·s^{-1}.
- $\dot{V}O_{2max}$ — Maximal oxygen uptake, L·min^{-1} or mL·kg^{-1}·min^{-1}.
- V_{vm} — The estimated minimal velocity that would drive $\dot{V}O_2$ reach $\dot{V}O_{2max}$, m·s^{-1}.
- v$\dot{V}O_{2max}$ — The minimal velocity that would drive $\dot{V}O_2$ to reach $\dot{V}O_{2max}$ as measured in an incremental test, m·s^{-1}.

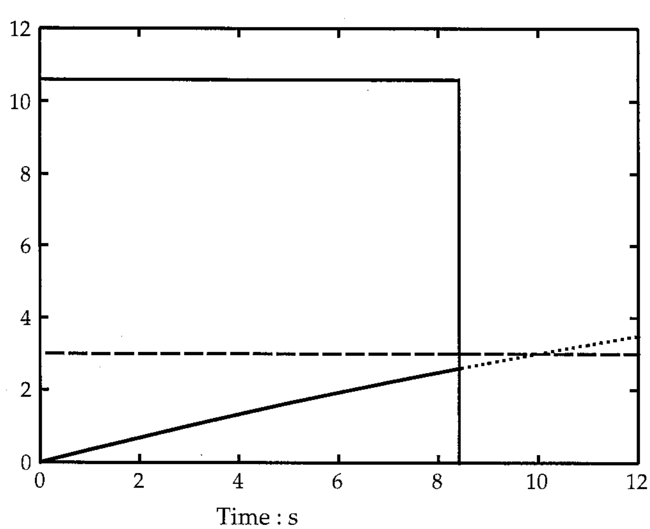

Figure A1—Energy supply/demand at very high velocity. This figure shows the energy demand at a constant velocity of 10.6 m·s^{-1}, for which endurance time is 8.42 s. The corresponding rectangular area shows the total work done, expressed in distance units. This area is comprised of aerobic work indicated by the area below the rising aerobic supply ($\dot{V}O_2$) line, and anaerobic work above this line. Exhaustion occurs before 10 s, the time $\dot{V}O_2$ would have taken to reach the equivalent of a CV of 3 m·s^{-1} shown by the *dashed line*.

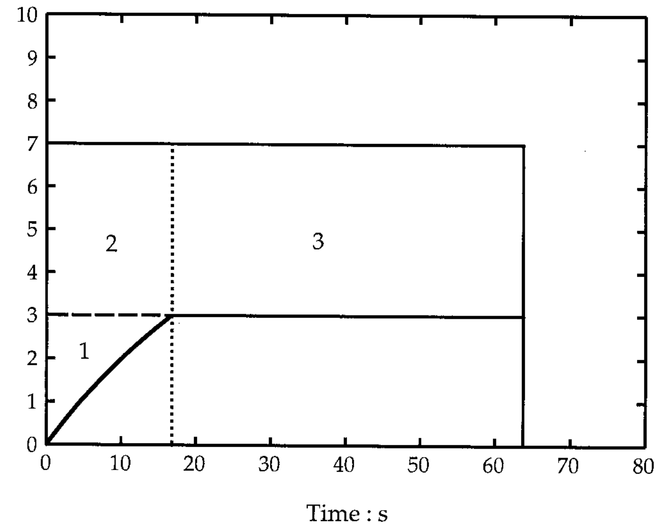

Figure A2—Energy supply/demand at high velocity. This figure shows the energy demand at a constant velocity of 7 m·s^{-1}, for which endurance time is 63.7 s. The corresponding rectangular area shows the total work done, expressed in distance units. This area is comprised of aerobic work indicated by the area below the aerobic supply line, which rises to reach the level equivalent to CV at 16.8 s, as shown by the *dashed line* at 3 m·s^{-1}, and thereafter remains constant at this level until exhaustion, and anaerobic work indicated by the areas marked 1, 2, and 3.

Linear control system. Morton (23,24) has conjectured that the maximal velocity that could be developed by a subject at any instant is controlled by the anaerobic capacity available at that instant, although it recognized that other causes of local fatigue may be involved. Specifically, this maximum, V_m, declines linearly from V_{max}, the maximal instantaneous velocity when fully rested and nourished, to CV, the critical velocity, as the available anaerobic capacity declines from its replete value α to zero.

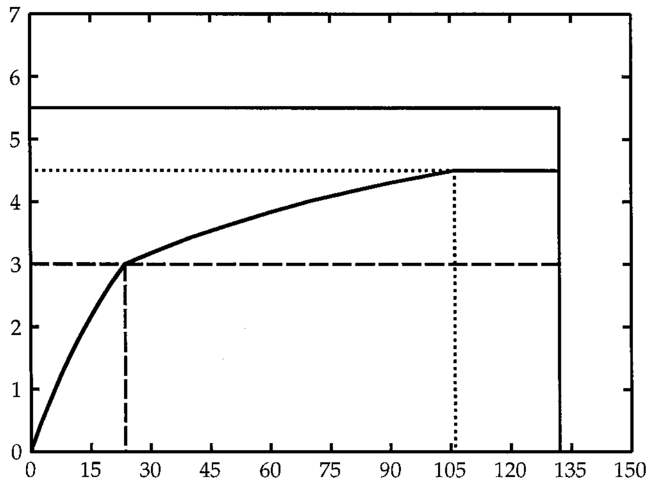

Figure A3—Oxygen uptake and energy supply/demand showing time at $\dot{V}O_{2max}$. For a constant velocity of 5.5 $m \cdot s^{-1}$, this figure shows oxygen uptake first rising rapidly to the equivalent of a CV of 3 $m \cdot s^{-1}$ by 23.65 s. This is followed by the slow component, driving it further, reaching $\dot{V}O_{2max}$ at an equivalent of 4.5 $m \cdot s^{-1}$ by 106.1 s. Thereafter, oxygen uptake remains constant at $\dot{V}O_{2max}$ until exhaustion at 132.1 s. Energy supply/demand features in this illustration are exactly analogous to those of Figure A2.

That is

$$V_m = CV + \left(\frac{V_{max} - CV}{\alpha}\right) AC \tag{1}$$

Exhaustion is precipitated and therefore endurance time determined when AC declines to such a value that V_m just equals the velocity required, V. Thus we must first determine AC as a function of time spent at the velocity V, substitute in equation 1 when $V_m = V$, and solve for t*, the endurance time at V.

Total endurance time at very high velocity. Upon commencement of exercise at velocity V, the aerobic supply, $\dot{V}O_2$, rises monoexponentially toward the oxygen requirement of V with a rate constant r_1 (1). That is

$$\dot{V}O_2 = V(1 - e^{-r_1 t})$$

It is conceivable that V may be so high that exhaustion occurs before $\dot{V}O_2$ reaches the equivalent of CV. That is for

$$0 < t^* \le t_c = \frac{-ln\left(1 - \frac{CV}{V}\right)}{r_1} \tag{2}$$

Where t_c is the time required for $\dot{V}O_2$ to reach the equivalent of CV.

The energy supply/demand relationship in such a case can be represented by Figure A1.

From this figure, we note that AC at t* is given by

$$\alpha - \left[Vt^* - \int_0^{t^*} V(1 - e^{-r_1 t})dt \right] = \alpha - \frac{V}{r_1}(1 - e^{-r_1 t^*})$$

Thus, using the linear control system of equation 1

$$V = CV + \left(\frac{V_{max} - CV}{\alpha}\right)\left(\alpha - \frac{V}{r_1}\left(1 - e^{-r_1 t^*}\right)\right)$$

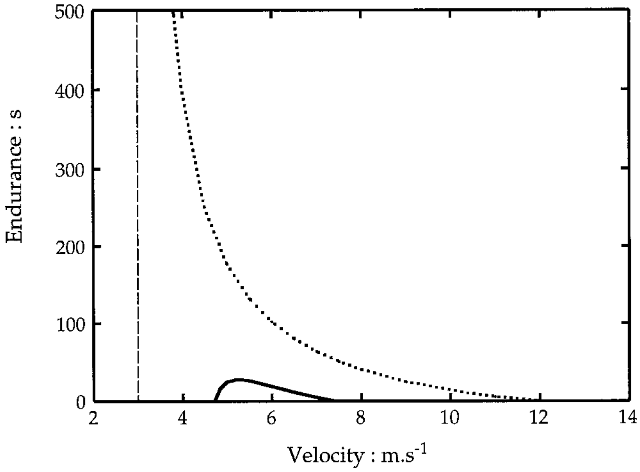

Figure A4—Model illustration. This figure shows graphical traces of the total endurance time (*dotted line*) and endurance time at $\dot{V}O_{2max}$ (*solid line*) as a function of velocity. The equations and parameter values are as given in the text, with CV = 3 $m \cdot s^{-1}$ shown in the figure by the *vertical dashed line*. Maximal endurance time at $\dot{V}O_{2max}$ is 27.8 s, occurring at V = 5.24 $m \cdot s^{-1}$ with corresponding total endurance time of 153.33 s.

which can be solved for t* to yield

$$t^* = \frac{-ln\left(1 - \frac{r_1\alpha(V_{max} - V)}{V(V_{max} - CV)}\right)}{r_1} \tag{3}$$

This of course only applies for $0 < t^* \le t_c$ given by equation 2, that is for a range of velocities V, where

$$V_{max} - \frac{CV(V_{max} - CV)}{r_1\alpha} \le V < V_{max}$$

Total endurance time at high velocity. Suppose the velocity required V, lies in the range

$$CV < V \le V_{max} - \frac{CV(V_{max} - CV)}{r_1\alpha}$$

which ensures $\dot{V}O_2$ reaches the equivalent of CV at time t_c prior to exhaustion. The energy supply/demand relationship in this case can be represented by Figure A2.

Area 1, expressed in meters, is given by

$$CV \cdot t_c - \int_0^{t_c} V(1 - e^{-r_1 t})dt = \frac{CV}{r_1} - t_c(V - CV)$$

More simply, area 2 is given by $(V - CV)t_c$, and we note that areas 1 and 2 sum to a constant, CV/r_1 independent of V.

Area 3 is given by $(V - CV)(t^* - t_c)$.

From Figure A2, we note that AC at t* is now given by

$$\alpha - \frac{CV}{r_1} - (V - CV)(t^* - t_c)$$

Thus applying the control system of equation 1,

$$V = CV + \left(\frac{V_{max} - CV}{\alpha}\right)\left(\alpha - \frac{CV}{r_1} - \left(V - CV)(t^* - t_c\right)\right)$$

Substituting for t_c from equation 2 and solving, yields

$$t^* = \frac{-\ln\left(1 - \frac{CV}{V}\right)}{r_1} + \frac{\alpha - \frac{CV}{r_1}}{V - CV} - \frac{\alpha}{V_{max} - CV} \qquad (4)$$

We note that if VO_2 was regarded as adjusting instantaneously to CV, then $r_1 \to \infty$ as is the case in the standard critical velocity model, then equation 4 reduces to

$$t^* = \frac{\alpha}{V - CV} - \frac{\alpha}{V_{max} - CV}$$

which is exactly the equivalent of the 3-parameter critical power model of Morton (24).

Endurance time at $\dot{V}O_{2max}$. Above CV, when the first exponential rise in $\dot{V}O_2$ is just complete or very nearly complete, the second, or slow component of oxygen uptake enters the model (1,14). $\dot{V}O_2$ will rise above the equivalent of CV but may not necessarily rise to reach $\dot{V}O_{2max}$. In either case, its contribution to the aerobic energy supply is assumed to remain at CV as conjectured by Barstow and Molé (1). $\dot{V}O_2$ may not reach $\dot{V}O_{2max}$ either because at high velocity the subject becomes exhausted too soon or because the $\dot{V}O_2$ equivalent of the exercise is below V_{vm}. In cases where $\dot{V}O_2$ does reach $\dot{V}O_{2max}$, the time taken in getting there must be subtracted from the total endurance time in order to obtain the time at $\dot{V}O_{2max}$.

Figure A3 shows the oxygen uptake kinetics, together with energy supply/demand features as shown in Figure A2.

We have already seen that the time taken for $\dot{V}O_2$ to reach the equivalent of CV is given by t_c derived from equation 1. From Figure A3, we note that for the slow component

$$(V_{vm} - CV) = (V - CV)(1 - e^{-r_2(t_{vm} - t_c)})$$

which yields

$$t_{vm} = t_c - \frac{\ln\left(1 - \frac{V_{vm} - CV}{V - CV}\right)}{r_2} \qquad (5)$$

Hence, because $t^* = t_{vm} + t_a$, equations 4 and 5 yield

$$t_a = \frac{\alpha - \frac{CV}{r_1}}{V - CV} - \frac{\alpha}{V_{max} - CV} + \frac{\ln\left(1 - \frac{V_{vm} - CV}{V - CV}\right)}{r_2} \qquad (6)$$

The limits of V between V_m and V_{max} for which $t_a = 0$ and between which $t_a > 0$, can be found by solving

$$\frac{\alpha - \frac{CV}{r_1}}{V - CV} + \frac{\ln\left(1 - \frac{V_{vm} - CV}{V - CV}\right)}{r_2} = \frac{\alpha}{V_{max} - CV}$$

It will be noted that this admits two solutions, the lower of which is a little greater than V_{vm}, and the upper somewhat less than V_{max}.

Illustration. Suppose $\alpha = 500$ m, $CV = 3\ m{\cdot}s^{-1}$, $V_{max} = 12\ m{\cdot}s^{-1}$, $V_{vm} = 4.5\ m{\cdot}s^{-1}$, $r_1 = 1/30\ s^{-1}$, and $r_2 = 1/90\ s^{-1}$. These values do not represent elite athletes. Equations 3, 4, and 6 are given by

$$t^* = -30\ln\left(1 - 1.852\left(\frac{12 - V}{V}\right)\right) \text{ for } 10.38 < V \le 12\ m{\cdot}s^{-1}$$

$$= -30\ln\left(1 - \frac{3}{V}\right) + \frac{410}{V - 3} - 55.56 \text{ for} 3 < V \le 10.38\ m{\cdot}s^{-1}\ t_a$$

$$= \frac{410}{V - 3} - 55.56 + 90\ln\left(1 - \frac{1.5}{V - 3}\right) \text{ for } 4.73 \le V \le 7.41\ m{\cdot}s^{-1}$$

These curves are depicted in Figure A4.