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Time in Human Endurance Models

From Empirical Models to Physiological Models

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Abstract

This article traces the study of interrelationships between power output, work done, velocity maintained or distance covered and the endurance time taken to achieve that objective. During the first half of the twentieth century, scientists examined world running records for distances from <100m to >1000km. Such examinations were empirical in nature, involving mainly graphical and crude curve-fitting techniques. These and later studies developed the use of distance/time or power/time models and attempted to use the parameters of these models to characterise the endurance capabilities of athletes. More recently, physiologists have proposed theoretical models based on the bioenergetic characteristics of humans (i.e. maximal power, maximal aerobic and anaerobic capacity and the control dynamics of the system). These models have become increasingly complex but they do not provide sound physiological and mathematical descriptions of the human bioenergetic system and its observed performance ability. Finally, we are able to propose new parameters that can be integrated into the modelling of the power/time relationship to explain the variability in endurance time limit at the same relative exercise power (e.g. 100% maximal oxygen uptake).

The limits of human physical endeavour is a topic guaranteed to raise animated discussion in any gathering. Athletics, and in particular running, is certainly one of the activities more frequently discussed in this light. Scientists too have investigated human physical achievements for over a century.

Table I gives the mens' world records for running various distances as at 30 January 1997. Most, although not all, of these are attributed to different runners, and the longest-standing records (800m and 1000m) are over 16 years old. Figure 1 has been created from these data and shows the reduction in average speed according to the logarithm of duration of the race.

It was obvious to the early investigators that such data must follow some pattern, the question being, just what pattern? This review traces the scientific attempts to answer this and other closely related questions over the course of this century.

Table I. Record times and speeds for male humans at various distances as at 30 January 1997

Distance	Time		Average speed	
	(h/min/sec)	(ln sec)[a]	(m/sec)	(km/h)
100m	9.84 sec	2.286	10.163	36.586
200m	19.32 sec	2.961	10.352	37.267
400m	43.29 sec	3.768	9.240	33.264
800m	1 min 41.73 sec	4.622	7.864	28.310
1000m	2 min 12.18 sec	4.884	7.565	27.234
1500m	3 min 27.37 sec	5.335	7.233	26.039
Mile	3 min 44.39 sec	5.413	7.170	25.814
2000m[b]	4 min 47.88 sec	5.663	6.947	25.009
3000m[b]	7 min 20.67 sec	6.088	6.808	24.509
5000m	12 min 44.39 sec	6.639	6.541	23.548
10 000m	26 min 38.08 sec	7.377	6.257	22.525
20km	56 min 55.6 sec	8.136	5.855	21.078
21.100km road	59 min 24 sec	8.179	5.920	21.312
21.101km	1h 00 min 00 sec	8.189	5.861	21.099
25km road	1h 13 min 55.8 sec	8.397	5.636	20.290
30km	1h 29 min 18.1 sec	8.586	5.599	20.156
42.195km	2h 06 min 50 sec	8.937	5.545	19.962
100km	6h 10 min 20 sec	10.009	4.500	16.200
200km	16h 32 min 20 sec	10.994	3.359	12.092
452.27km	48h	12.060	2.617	9.421
1023.2km	144h (6 days)	13.159	1.973	7.103

a The natural logarithm of time in sec.

b These speeds are close to $v\dot{V}O_{2max}$, the minimal speed eliciting maximal oxygen consumption ($\dot{V}O_{2max}$).

1. Empirical Methods

In 1906, Kennelly[1] investigated the velocity/time relationship for running by analysis of the world records at that time. In this article, he developed an approximate law of fatigue for humans and horses and discovered a relationship between the speed (v_t) that can be maintained over a time (t), and t itself:

$$v_t = k/t^n \qquad \text{(Eq. 1)}$$

in which k is a constant for the type of work and n is an exponent which varies between 0.125 and 0.111 (1/8 and 1/9). Thus, since v_t = race distance in metres (d)/t:

$$\ln t = 8/9 \cdot \ln d - 1.2307 \qquad \text{(Eq. 2)}$$

and doubling the race distance leads to an increase of 118% in race time. Kennelly advised athletes who were keen to break world record times to attempt those points situated above a line tracing the time versus distance relationship (equation 2). Kennelly also noted the similarities between the equations formulated for humans and those for horses, taking care to distinguish between walking (for humans and horses), trotting and galloping (running for humans).

Kennelly reached similar conclusions about the velocity/time relationship 20 years later, this time taking into account gender and the type of locomotion. His investigations included cycling, skating, running, rowing, hurdling, walking and swimming (freestyle), and even automobile racing.[2]

After Kennelly, Meade[3] was the next to examine the limits of human performance. He emphasised the reliable nature of world records and suggested the possibility of using them to deduce human physiological characteristics. By establishing the relationship between pace (time per mile) and distance, he noticed that performance in longer races [over 16.1km (10 miles)] was relatively superior (i.e. the data points fell below the speed/distance curve). Meade explained this by the fact that these races were run by professionals who were highly moti-

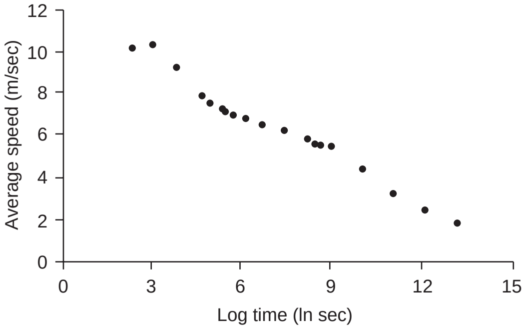

Fig. 1. Relationship between running speed and natural logarithm of race duration in sec for male world records in running events, as on 30 January 1997.

vated by money. He deduced, therefore, that amateur records had a large potential for improvement and speculated on the usefulness of a mathematical analysis applied to predict future performance, but went no further in this article which was exploratory rather than explanatory. However, in 1934, Meade (unpublished observations) commented on the form of the speed/distance relationship, stating that it was probably not logarithmic and did not follow the formula given by Kennelly.

The mathematical analysis of the power/duration curve (fig. 2) was next developed by Grosse-Lordemann and Müller,[4] who resumed Kennelly's work on the world records of their day. In 1937, Grosse-Lordemann and Müller studied 6 individuals who performed exercise for up to 120 min. Figure 2 shows a typical power/duration curve for an individual who has undertaken 6 endurance trials on the cycle ergometer. They derived empirical equations from their results, only one of which was subsequently used 25 years later by Tornvall:[5]

$$\log t = a \cdot \log P + b \qquad \text{(Eq. 3)}$$

or after rearrangement:

$$t = 10^{(a \cdot \log P + b)} \qquad \text{(Eq. 4)}$$

where P is the sustained power (in watts), t is the total time during which this power is sustained (sec), a and b are estimated as –3.04 and 10.01, respectively, and are obtained empirically by a least-squares analysis of 8 experimental points by taking the logarithm of P as an independent variable. Parameters a and b varied greatly between individuals, even though Grosse-Lordemann and Müller did not give them any physiological significance.

Müller put forward another empirical equation in 1938 (referred to in Purdy[6]):

$$\log W_{tot} = cP + d \qquad \text{(Eq. 5)}$$

where W_{tot} is the total work (joules) and c and d are 2 constants equal to -1.92×10^{-3} and 5.50, respectively, which are obtained under the same conditions as constants a and b in equations 3 and 4. Equation 5 can be written:

$$t = 10^{(cP + d)}/P \qquad \text{(Eq. 6)}$$

This empirical approach to the endurance model (the relationship between speed or racing pace and the distance or racing time) was followed by work by Francis.[7] He examined the plot of speed against the logarithm of racing distance and attempted to fit it to a hyperbolic curve. This curve was satisfactorily predictive for distances between 400m and 19km. The hyperbolic equation calculated by Francis from experimental points was as follows:

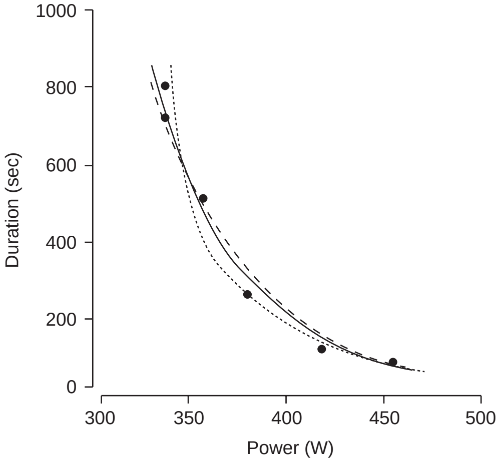

Fig. 2. Power/duration relationship for cycle ergometry. For each power demand/endurance time combination, a point is defined. Three different fitted curves are shown for illustrative purposes.

$$(\log d - 1.5) \times (v - 3.2) = 6.081 \quad \text{(Eq. 7)}$$

where d is the distance in metres and v is the speed in m/sec. The horizontal asymptote of the hyperbola represented a speed sustainable without fatigue, which Francis then identified as being a 'dog trot', a speed of 3.2 m/sec (11.5 km/h) for a 'perfect' runner. Francis assumed that this speed could be maintained indefinitely by disregarding the necessary sleeping hours. This was perhaps the first suggestion of a model incorporating human bioenergetic characteristics, but many years were to elapse before the potential of using these characteristics to predict individual performance began to be realised.

2. Models of the Power/Distance/Speed/Time Relationship Used to Characterise Athletic Endurance

The analysis of world records gave valuable general information on the limits of human performance. However, this analysis did not allow the endurance for an individual to be estimated. Endurance is defined as being the capacity to sustain a given power for the longest time possible (or sustain the greatest power over a given time), a quality which could be related to the bioenergetic characteristic already known at the time as the maximal oxygen deficit. Time is measured until exhaustion, and according to Edwards et al.[8] exhaustion is defined as failure to maintain an imposed power and is brought on by fatigue. It is important to realise that fatigue may be physiological or psychological in origin, and can be regarded as the inability to perform maximally.

Henry and Farmer[9] adopted the notion of endurance by defining an endurance or 'drop-off' index by comparing the time required for running 220 yards (201m) and 70 yards (64m):

$$\text{endurance index} = \frac{\text{time for 220 yards}}{\text{time for 70 yards}} \quad \text{(Eq. 8)}$$

This index correlated well with the subjective estimate of the state of training (untrained, slightly trained, well trained, extremely well trained) in 18 sports students as well as the presumed state of physical condition (poor, fairly good, good, excellent). Therefore, Henry and Farmer concluded that since this experiment had been specifically designed to assess the endurance of the participants (and not extrapolated from the records) endurance could be assessed using the subjective judgement of the participants regarding their training status and physiological condition.

Five years later, Henry[10] examined the historical evolution of the 1 mile record since 1865. He stated that most 1 mile records could be fitted to an experimental curve obtained by linking the record time for 1 mile with the year in which it was achieved. Despite this, he did not venture to predict the date upon which the 4 min barrier would be broken (3 min 59.4 sec by Roger Bannister in 1954). He supposed that this barrier would reveal the physiological limits, without specifying what they could be.

The same method of evaluating human performance and endurance limits by examining the evolution of world records was adopted by Lietzke,[11] who analysed the relationship between pace (time per 100m) and distance for swimming, walking and running records. He considered that all of the points situated below the curve of pace versus distance could easily be beaten, and by calculating the required gain in pace and dividing by the pacing distance over 100m he obtained a time margin that it was possible to gain over each of the considered distances. Lietzke noticed that the maximal speed on the distance/time relationship curve (using a logarithmic scale) was achieved at 15 sec, being a speed of 35.91 km/h which, today, is still considered to be correct, even if the maximal speed is closer to 40 km/h. Lietzke therefore deplored the absence of official 100m and 200m races which would allow this maximal speed to be seen. Lietzke pointed out that the analysis of world records for racing on foot, the basis of the human endurance curve, should begin with the 150m because for shorter distances the reaction and acceleration times at the start slightly lower the average speed. He noticed that the curve follows a negative straight line slope to 140 sec racing time (and a speed of 26 km/h) over a distance of roughly 1000m, and thereafter the curve follows a more gentle slope with a lower loss of speed with lengthening of exercise duration. This is why this distance of 1000m delimits long and short races.

Lietzke did not speculate upon the fact that from a starting point of 2 (or 3 depending on the author) minutes of exercise aerobic metabolism becomes predominant, which explains the plateauing of the speed in spite of the lengthening of exercise duration. Lietzke attributed the decreased loss of speed with longer duration to ‘second wind’, and noted that from 12.5 miles (20.1km) and above the speed curve decreases again, which we can now explain by depletion of glycogen and the necessary passage to lipolysis, a less favourable process of oxidative phosphorylation.

Lietzke related the distance run (d) to time (t) and found an almost linear relationship of the logarithms:

$$\log d = k \cdot \log t + \log a \qquad \text{(Eq. 9)}$$

where k is the slope and log a is the intercept of the straight line. Equation 9 can therefore be written as:

$$d = at^{k} \qquad \text{(Eq. 10)}$$

or

$$t = (d/a)^{1/k} \qquad \text{(Eq. 11)}$$

$$\text{Speed } (v) = d/t = at^{k}/t \rightarrow at^{k-1} \qquad \text{(Eq. 12)}$$

From equations 11 and 12:

$$v = a^{1/k} d^{(k-1)/k} \qquad \text{(Eq. 13)}$$

A ‘constant of exhaustion’ $\kappa = (k - 1)/k$ can therefore be calculated using equation 10. This constant of exhaustion for running (–0.239 for women and –0.009 for men) gives a quantitative measure of the delay of fatigue appearance. This therefore represents a second quantification of human endurance after that of Henry and Farmer.[9]

Two years later, Lietzke[12] examined the speed/time relationship curve plotted on a double-logarithmic scale to obtain 2 straight segments on either side of the time taken to run 1 mile (removing the 100m time, since the short sprint alters the average speed too much). He therefore distinguished the equivalence of records situated on the lines (the ‘best efforts’) from those for which he predicted a rapid improvement:

- a gain of 2.9 sec over 800m (1 min 43.7 sec rather than 1 min 46.6 sec)
- a gain of 4.2 sec over 1000m (2 min 15.3 sec rather than 2 min 19.5 sec)
- a gain of 29.9 sec over 10 000m (28 min 24.3 sec rather than 28 min 54.2 sec).

Meade[13] criticised Lietzke’s[12] work by denouncing his lack of originality compared with the pioneer Kennelly[1] and advised Lietzke to take into account the historical and cultural contexts surrounding the achievement of records at various distances, many among them having rarely been attempted. Meade concluded that a definite relationship between speed and racing time was only reliable if it was based upon historical knowledge, practical experience in each of the racing distances and, most of all, the use of physiological considerations rather than a simple statistical analysis. He therefore favoured the contemporary work of Henry[10,16] to those of Lietzke. Some years later, Péronnet[14] made similar remarks in response to an article by Whipp and Ward[15] speculating upon the future increase of female and male marathon performances by extrapolating female performances starting from their evolution around 1980. It was clear that the rapid evolution of female marathon performances came from the fact that this distance has only been part of the Olympic programme since 1984 (Los Angeles). All agree that the ultimate goal is to give a biochemical explanation for all aspects of the speed/time curve.

It was the later work of Henry[16] that marked the beginning of a physiological approach to the relationship linking power and duration of exercise. We will continue this theme in section 3, which discusses theoretical models of the power/time relationship based on human bioenergetic characteristics.

Tornvall[5] utilised Grosse-Lordemann and Müller's[4] formula to compare it to experimental values from the laboratory. To this effect, he devised an experiment on 28 volunteers who accomplished between 2 and 5 exhaustive exercises on different days. These exercises exhausted the participants in 1 to 18 min. By plotting the straight line between the time (logarithmic scale) and the amount of work in kilopond-metres/min [1 kilopond-metre (kpm) = 9.8066J; 1 kpm/min = 0.167W] he could determine the slope of this line, conveying the balance between anaerobic and aerobic capacity as an index of fatiguability, but without going any further into bioenergetic-type explanations.

At the same time, Craig[17] endeavoured to evaluate and predict the world records for running and swimming by relating the record curve for distances to the date on which they were achieved (from 1920 to 1963). He claimed a larger improvement for the longer races (>800m for swimming and >1 mile for running). He instigated this study to follow Meade's[13] criticisms of the work of Lietzke.[11,12] Craig considered it impossible to predict performances without having seen the evolution of these records as a whole over history, regarding the prediction of a single achieved current record as 'aristocratic statistics' (a personal comment by Karpovich to Craig). Craig therefore compared the speed/time curves of the records for running in 1920 and 1961. He noticed that the shapes of the 2 curves were similar, with a rapid decrease in speed for distances lasting less than 5 min and with a more gentle slowing down from then onwards. He did not consider this to be surprising in the sense that the establishment of world records requires human beings with the same physical and physiological qualities. The progress achieved in the 40 years reflects improved racing and training techniques, which are apparently the same for all racing distances since the form of the curve is similar over a 40-year interval. Craig[17] recommended comparing a runner's own curve to that of the world records to assess quality of speed and endurance and direct him/her towards longer or shorter distances. The ratio of performance to world record, at any distance, could be considered as an index of relative performance.

These speed/time relationships only consider average speeds. Sustaining a constant racing speed is regarded as the optimal strategy for breaking a record for distances greater than 291m (distinguishing the long and sprint distances), as demonstrated by Keller.[18] Keller studied the optimal choice of racing speed to break a record. The constraint of the optimisation equation is the achievement of the shortest time possible t over a given distance d. The variable describing the energy flux depends on the runner's maximal oxygen uptake ($\dot{V}O_{2max}$), together with an initial quantity of available energy. Thus, the problem to be solved is finding a racing speed compatible with the energy stores, their flow renewal, and the minimal duration of exercise.

This theory is based on Newton's second law and the calculus of variations which determines an optimal strategy to be used for taking the shortest possible time over a given distance according to equations 14 and 15 for distances <291m and >291m, respectively:

$$d = F\tau^2[t/\tau + e^{(-t/\tau)} - 1] \qquad \text{(Eq. 14)}$$

$$v^2(t) = \sigma\tau + [v^2(t_1) - \sigma\tau]\, e^{-2(t_2 - t_1)/\tau} \qquad \text{(Eq. 15)}$$

where F is the constant maximal force that the runner can exert; σ is the energy equivalent of maximal aerobic power; τ is a constant of proportion for the forces of external resistance to motion; and t_1 and t_2 the times between which speed v is constant over the racing interval. The bioenergetic parameters F, σ and τ and the split times t_1 and t_2 may be determined by comparing theoretical predictions with the world records.

In 1976, Ryder et al.[19] analysed the improvement of pace in foot-running performance from 60 yards to 30km between 1926 and 1976. For the 100m, he claimed a gain of 0.6 m/min of exercise (10 cm) per year, and for long-distance races an improvement of 0.9 m/min/year.

In 1977, Frederick[20] published a statistical analysis of the pace/time model for 62 international-level long-distance and semi–long-distance runners.

He established a linear regression between the racing pace (in min/mile) and the logarithm of the accomplished distance of the race. The slope is considered to be a 'coefficient of fatigue, f,' an expression of the rate of speed decrease according to race duration. He observed differing values of f according to sporting speciality, which fell into 3 categories: 1500m, 5000m and the marathon. By examining the runner's factor f, it would be possible to orientate them to a racing distance: f = 1.0 ± 0.09 for runners of the 1500m; f = 0.699 ± 0.021 for runners of 5000 to 10 000m; and f = 0.619 ± 0.02 for marathon runners. Frederick's idea of a fatigue coefficient is very similar to that of Coleman and Rumball.[21] However, none of these authors gave any physiological significance to this factor f, which is considered to be a complex association of physiological, psychological and morphological factors. Moreover, the effects of training on the value of f are slight. For example, Craig Virgin (world cross-country champion in 1980) had a change in factor f from 0.572 to 0.570 between 1972 and 1976.

Morton[22,23] undertook an examination of the time trend in world records over distances from 100m to the marathon. An asymptotic exponential regression trend was fitted to the data, which clearly demonstrated that ultimate time limits could be estimated for all such distances for the extended future. Ultimate times of 9.15 sec for the 100m, 3 min 4.15 sec for the 1500m and 23 min 40.94 sec for the 10 000m were predicted. A complete table of these limits and progress towards them can be found in Noakes.[24] A cross-sectional study of these limits using the model of Keller[18] enabled ultimate estimates of the maximal propulsive force, maximal aerobic power and anaerobic capacity to be obtained. The ultimate male superathlete of the extended future is predicted to be able to exert a maximal accelerative force of 15.0 $m/s^2/kg$ of bodyweight, have a maximal aerobic power of 154 ml/kg/min and a useable anaerobic capacity of 0.14 L of O_2/kg of bodyweight.

Harman et al.[25] also proposed an endurance index calculated from the difference between the absolute and relative power scale (in watts and maximal power percentage respectively). The aim of this study was to quantify interpersonal differences in endurance. This indicator of power therefore needed to be correlated to physiological characteristics such as muscular typology and $\dot{V}O_{2max}$. It was an experimental approach with each participant undertaking an exhaustive test on a bicycle ergometer at 36, 45, 54, 63, 72 and 81% of the maximal power measured in an all-out exhaustive effort of 3 to 5 sec (measured 4 times at intervals of 20 min). The individuals who did not last for 5 min at 36% of maximal power were tested at a lower power. Additional power levels were introduced so that every individual had 9 experimental points to form their own personal curve. The test was stopped as soon as the pedalling speed dropped by 3% of the imposed frequency for 7 sec, this 7 sec being subtracted from the total exercise time. The relationship linking power with exercise time was curvilinear:

$$t = a(P_{sc})^b \qquad \text{(Eq. 16)}$$

where t is the endurance time (in minutes); a and b are empirical constants; and P_{sc} ('scaled power') is the relative power as a percentage of maximal power.

Each individual therefore has a pairing of absolute and relative power, P_{sc} being the relative power. Harman et al.[25] showed that the dispersion of endurance time for a given absolute power is actually increased by utilising scaling to relative power. This showed that within a group of individuals, relative power was a poorer predictor of endurance time than absolute power. This initially surprising result could signify that the interpersonal differences in endurance are rather more linked to anaerobic capacity at the onset of lactate accumulation, or critical speed,[26] as was also pointed out by Vandewalle's group in 1996.[27] Harman et al.[25] plotted the endurance time graphs according to power expressed as a percentage of the maximum power. By definition these curves coincide at 100% of the maximal power, even though they diverge at a lower relative power level. Harman incorporated the features of endurance into a factor F (individual scaling factor) which is calculated by using the differences between the values on the absolute and relative power scales in order for the endurance time to be similar.

$$P_{sc} = 100 - F(100 - P_{Pmax}) \quad \text{(Eq. 17)}$$

where P_{sc} is the scale power calculated from P_{Pmax}, the relative power expressed as a percentage of the absolute maximal power measured during a test lasting between 3 and 5 sec. The quantity F represents an individual scale factor which expresses a fatigue index. When $P_{Pmax} = 100$ or if $F = 1.00$, then $P_{sc} = P_{Pmax}$: when $P_{Pmax} < 100$ or if $F > 1.00$ then $P_{sc} < P_{Pmax}$.

We can also calculate F by measuring the relative movement of the point on the power axis (x) which attains the same exercise duration:

$$F = (100 - x_{new})/(100 - x_{old}) \quad \text{(Eq. 18)}$$

where x_{old} is the old co-ordinate for a given time value on the power axis as an absolute value and x_{new} is the new co-ordinate for the same time value on the power axis in relative values (as percentage of P_{Pmax}) when $P_{Pmax} = 100$.

3. Theoretical Models of the Power/Time Relationship Based on Human Bioenergetic Characteristics

3.1 Early Attempts to Attribute Physiological Meaning to Parameters of Empirical Models

In 1954, Henry[10] introduced a preliminary physiological explanation for the characteristics of the speed/time relationship described in section 2. His explanation was the exhaustion of various fuel reserves for transformation of chemical into mechanical energy. These reserves invoked alactic and lactic anaerobic metabolism according to the duration of exercise (manifested by the oxygen debts both alactic and lactic in origin), then aerobic metabolism from glycogen reserves, from fat and eventually from protein. In proposing a more physiological explanation of speed decrement according to race time, Henry proposed a rate constant a_n for each section of the general speed/time relationship curve plotted on a double-logarithmic scale:

$$dy/dt = a_1e^{-k_1t} + a_2e^{-k_2t} + a_3e^{-k_3t} + a_4e^{-k_4t} + a_5e^{-k_5t} \quad \text{(Eq. 19)}$$

a_n being in m/sec and k in sec^{-1}. The range of reduction in speed can be estimated from the following 4 values of the coefficient a_n, which represent in a general way an energy debit causing a loss of speed because of the depletion of energy reserves:

- a_1 represents the loss of speed caused by depletion of alactic anaerobic reserves = 4.80
- a_2 represents the loss of speed caused by depletion of lactic anaerobic reserves = 1.80
- a_3 represents the loss of speed caused by depletion of glycogen reserves = 2.96
- a_4 represents the loss of speed caused by depletion of fat reserves = 3.54
- a_5 represents the loss of speed caused by depletion of the (negligible) protein reserves.

Hill[28] had noticed that oxygen uptake ($\dot{V}O_2$) had not reached a truly steady state after 4 min of exercise at 16 km/h, but that the oxygen debt increased. He therefore sensed that such exercise could not be pursued indefinitely. He advanced the notion of a ‘reasonable time limit above which an oxygen deficit appears as time passes, the blood lactate accumulates, the cardiac frequency and corporal temperature increase, the respiratory quotient persists higher than 1 and the ventilatory debt increases’. Thereafter glycogen reserves run out, glycaemia plummets and muscular stiffness and pain appear, here describing his own symptoms. He therefore concluded that all these factors determine a runner’s endurance. A maximal steady-state speed is that which, according to Hill, it was possible to maintain for half to three-quarters of an hour. This speed does not induce hyperthermia nor hypoglycaemia, contrary to what Hill had claimed for marathon runners. He demonstrated $\dot{V}O_{2max}$ at a value of 4.41 L/min achieved in 4 min and maintained for 6 min by a rower weighing 83.5kg. It is highly unlikely that this relative $\dot{V}O_2$ value, equal to 49 ml/min/kg, is representative of maximal performance attained at that time. This value is somewhat below the $\dot{V}O_2$ (73 ml/min/kg) necessary for running 2 miles at 21 km/h, the world record at the time, using the standard oxygen cost of running of 210 ml/min for a 60kg human (3.5 ml/min/kg per km/h of speed) calculated by di Prampero.[29] In 1927, world record holders had never been studied in the laboratory.

Hill[28] concluded that the highest speed that could be sustained in a given fixed time was determined by the energy supply and the metabolic reserve, proposing 3 possibilities for a runner whose $\dot{V}O_{2max}$ equals 4 L/min and whose maximal accumulated O_2 deficit (MAOD[30]) is 16L:

(i) If the runner could use all 20L of oxygen in 1 min (equivalent to a $\dot{V}O_2$ of 285.7 ml/min/kg for a bodyweight of 70kg), he/she could run extremely fast at 81.6 km/h.

(ii) If the same runner ran for 2 min and therefore used a total of 24L of oxygen for a $\dot{V}O_2$ of 171 ml/min/kg, this could give a speed of 48.9 km/h, getting closer to (record) speeds achieved in sprints.

(iii) If our model runner opted for a 7 min race, this would allow use of 44L of oxygen ($\dot{V}O_2$ 89.8 ml/min/kg) and a corresponding speed of 25.7 km/h. In 7 min this would give a distance of 2993m. Thus a 3000m time of just over 7 min is predicted.

This demonstration allowed Hill to show that oxygen deficit makes a gradually lower relative contribution as racing time increases. Thus, according to Hill, the relationship between racing speed, distance and time involves: (i) the oxygen consumption necessary for running the distance in the given time; (ii) the runner's effective $\dot{V}O_2$ during the race; and (iii) the oxygen debt contracted by the runner during the event (in fact, the deficit according to Medbo et al.[30]):

$$E = S + Rt \qquad \text{(Eq. 20)}$$

where E is the energy spent, S is the energetic capacity or supply and R is the energy debit rate during exercise.

Hill[28] therefore formulated the relationships linking speed, distance and time with R and S by means of the following equations:

$$v = S/(tB) + (R - A)/B \qquad \text{(Eq. 21)}$$

$$d = S/B + t(R - A)/B \qquad \text{(Eq. 22)}$$

where A and B are individual parameters depending on the runner's physiological characteristics, A being the $\dot{V}O_2$ at rest [not very variable among participants, equal to an average of 1 metabolic equivalent (MET), i.e. 3.5 ml/min/kg] and B is the energy debit rate for a given speed expressed as the $\dot{V}O_2$ equivalent to a given speed (or the racing economy according to the definition given by Daniels et al.[31]). For short distances (limit not specified), S is small and R large; for long distances, S is larger and R is smaller.

This relationship was further examined by Sargent[32] over a stretch of 120 yards run at speeds that were strictly controlled by a runner who, it was declared, could run short distances as well as he could run long distances. The $\dot{V}O_2$ during rest was measured standing up after lying down for 30 min (an early example of the method of retroextrapolation revisited in the 1980s by Léger and Boucher[33]). It was not the kinetics of oxygen uptake but the total volume (oxygen debt) that was examined *a posteriori* with the runner having held his breath during the 120 yard race and breathing until his $\dot{V}O_2$ returned to its rest value. At a speed in excess of 5.5 m/sec (19.8 km/h), the speed/$\dot{V}O_2$ relationship became exponential:

$$\dot{V}O_2 = k \cdot v^{3.8} \qquad \text{(Eq. 23)}$$

At the time, 5.5 m/sec was estimated to be the runner's maximal aerobic speed (speed above which $\dot{V}O_2$ did not rise despite an increase in racing speed), even if this concept had not then been formulated. By knowing the runner's accumulated maximal oxygen deficit, the equivalent cost of oxygen of the race at each speed and the runner's $\dot{V}O_{2max}$, it was possible to identify the distance limit covered at racing speed. The runner could then cover 1708m in 5 min at a speed of 5.69 m/sec (20.5 km/h). This time probably constituted the time limit (t_{lim}) at $\dot{V}O_{2max}$. Sargent concluded that with this method it was possible to calculate a distance limit at the athletes' given speed and, in the same way, to estimate their performances over regulated distances between 300 yards (274.2m) and 2 miles (3218m).

Hill's[28] hypothesis, according to which the oxygen debt was caused by a delay of oxidation in response to muscular activity, thus explaining a quantitative relationship between oxygen debt and the production of lactic acid, was generally accepted in the early 1930s. However, Margaria et al.[34] proposed the possible existence of an alactic debt and noted that the accumulation of lactic acid only occurs beyond an exercise power equal to about two-thirds of that corresponding to the $\dot{V}O_{2max}$. They did not mention a time limitation of exercise at a power corresponding to the onset of lactate accumulation, but were rather more interested in post-exercise lactate kinetics in relation to that of $\dot{V}O_2$ to identify alactic and lactic debts.

3.2 Transition to the First Real Physiological Models for Human Endurance

Scherrer et al.[35] studied the influence of load and imposed frequency of local dynamic muscular work, as well as the influence of circulatory occlusion, to specify 'a few valuable rules for the muscular work of Man'. The work consisted of lifting a load to a given height (for the biceps, the brachial triceps and the quadriceps). A 'critical power' for working was established together with the notion of work capacity. The research which lead to this result used the 'energameter' at Bidou's constant power (1947, quoted by Monod[36]) in which resistive work added itself to active work. Scherrer et al.[35] has named the 'threshold of local fatigue' as the moment where the work can no longer be continued at the initial power, the t_{lim} as the duration for which it had been maintained and work limit (W_{lim}) as the total work carried out. The notion of critical power was based on a simple linear relationship linking work and time limits, expressed as:

$$W_{lim} = a + b \cdot t_{lim} \quad \text{(Eq. 24)}$$

where a and b are constants whose physical meaning can be deduced from experiments with and without blood flow occlusion, such that factor a is a reserve of W_{lim} measurable under occlusion and factor b is the maximum rate of reconstitution of the energetic potential of muscular contraction. As long as working power is less than or equal to factor b, the work may be continued over a very prolonged period of time. The critical power is therefore equal to the value of factor b. Starting with equation 24 which relates W_{lim} to t_{lim}, the power (P) at which the work is realised is:

$$P = W_{lin}/t_{lim} \quad \text{(Eq. 25)}$$

and after substitution, we obtain:

$$t_{lim} = a/(P - b) \quad \text{(Eq. 26)}$$

Equation 26 shows that t_{lim} is therefore an inverse function of the difference between the power of the imposed work and factor b, which is equivalent to the maximum power of restoration of muscle function. In other words, there is a hyperbolic relationship between t_{lim} and power.[36] In the studies of Scherrer et al.,[35] t_{lim} was between 1 and 20 min with a few values up to 1 hour.

This relationship lends itself particularly well to estimating exercise t_{lim} between 4 and 30 min. This model and experimental approach were then applied to general dynamic work involving more than two-thirds of the total muscular mass, for example running on foot or pedalling a cycle. We will return to the concept of critical power applied to general dynamic exercise at the beginning of the 1980s by Moritani et al.[37] and then Vandewalle et al.[38] for pedalling exercises, by Hughson et al.[39] and then Lechevalier et al.[40] for running on foot, and by Wakayoshi et al.[41,42] for swimming. We will also see that this model allows us to explain the observed relationship between t_{lim} at $\dot{V}O_{2max,}$ $\dot{V}O_{2max}$, and the speed at the onset of lactate accumulation (which is very close to the critical speed determined according to the model of Scherrer et al.[35]). In particular, in section 3.4, we investigate interpersonal variability in t_{lim} at the speed corresponding to $\dot{V}O_{2max}$ ($v\dot{V}O_{2max}$).

In 1960, and then in 1980, Wilkie[43,44] proposed that these purely empirical equations had been devised over durations that were too long to allow an accurate physiological interpretation. He therefore suggested the following equation:

$$P = E + (A/t) - E\tau[1 - \exp(-t/\tau)]/t \quad \text{(Eq. 27)}$$

where P is the required power, the first term E of the equation represents the maximal aerobic power (273W, for example), and the term A is the work that can be accomplished from the anaerobic energy sources (16kJ, from the same example). The third, and more complex, term explains the fact that aerobic power does not immediately attain its maximal value, a time constant τ of about 10 sec being necessary. This would be negligible for exercise of several minutes or longer, but was important for exercises of up to 3 min. Wilkie demonstrated little rigour or interest in the elaboration of such models. If the modelling has a degree of physiological reality, then its characteristics – despite simplicity – are not to be neglected. We must therefore question the predictive value of the model (or how the equation conforms to the experimental data) and whether its form suggests a regulating mechanism and its parameters correlate with other determinable quantities.

For example, although the value of the maximal aerobic power (273W) is compatible with a $\dot{V}O_{2max}$ of about 4 L/min, and an anaerobic capacity of 16kJ with the energetic equivalent of lactic acid (where 1 mmol/L accumulated = 3ml of O_2 consumed/min/kg bodyweight[45]), the time constant τ seems too brief in comparison with the kinetic delay in adjustment measured for $\dot{V}O_2$, which was more like 30 sec.[46] Wilkie's explanation was that the intramuscular oxygen reserves could compensate for this difference at a local level. Wilkie[43] established human power and time limits as follows (1hp = 735.5W):

- for isolated movements, power is limited to less than 6hp by the muscle's own power and by the difficulty of coupling each muscle to an appropriate weight
- for brief durations of effort (0.1 to 5 min), power is limited to 2 to 0.5hp by the internal chemical resources of the muscles
- during work at steady state between 5 and ≥150 min, power is limited to 0.5 to 0.4hp
- for long durations of work (the whole day), power is reduced by fatigue and is estimated at 0.2hp
- these values are for exceptional athletes – individuals who are healthy but untrained only provide 70 to 80% of the indicated power.

In an article investigating the limits of human performance in keeping with energy production, Frederick[47] proposed a relationship between distance and t_{lim} calculated from the world records of the time. To establish the relationship between distance and t_{lim}, Frederick used the following distances and activities: 100, 200, 400, 800, 1500, 5000, 10 000m and marathon (running on foot); 100, 200, 400, 800 and 1500m (swimming); 100, 200, 400, 800, 1500, 5000 and 10 000m (skating); 1, 5, 10 and 20km and the 1 hour record (cycling). The sports and distances had in common the fact that they were dependent on respiratory and circulatory systems. It was necessary to assume that the athlete was exhausted on termination of the distance and that the time could be taken as t_{lim} for the respective distance. The speed/time relationship curve for each sport has an almost constant slope for short distances, followed by a smaller but again almost constant decrease in speed for distance covered in >240 sec. The relationship proposed by Frederick is:

$$\log v = a + b \cdot \log t \quad \text{(Eq. 28)}$$

where a and b are constants.

In 1966, Lloyd[48] analysed the world records for running on foot by testing Hill's[28] model (equations 21 and 22), following his analogy with the concepts of economy: S representing energy reserves (the stock) and R, the income according to the distances run. Using this model, and by examining changes in its parameters over time, Lloyd predicted that times in all races up to 10 000m would be reduced by about 5.5%, and in longer races by about 7.5%, by the year 2000.

In the same year, Ettema[49] re-examined the model of Scherrer et al.[35] Ettema emphasised the paradox that, according to Frederick's[47] model (equation 28), short distance performances would be overestimated when calculated using data obtained over long distances. He therefore proposed a relationship between distance and t_{lim} rather than a speed/time relationship. He considered it easier to interpret the curve linking distance to record time (t_{lim}), i.e.:

$$d = a + b \cdot t_{lim} \quad \text{(Eq. 29)}$$

where a is the distance in metres which could be run on oxygen reserves and the energy supplied by anaerobic metabolism, and maximal speed b (in m/sec) is interpreted as the rate of reconstitution of these reserves by aerobic metabolism. Ettema noted the equivalence of his equation to that proposed by Scherrer and Monod[50] for dynamic local work of a group of muscles. From equation 29:

$$v = a/t_{lim} + b \quad \text{(Eq. 30)}$$

and:

$$t_{lim} = a/(v - b) \quad \text{(Eq. 31)}$$

where v is the speed in m/sec, t_{lim} is in sec and a and b are constants whose values differ according to the section of the curve studied, i.e. to the range of t_{lim} and thus the relative contributions of aerobic and anaerobic metabolism.

The t_{lim} at a particular speed would then depend, according to Ettema,[49] on the difference between that speed and the possible maximal speed for reconstitution of the energy stores by oxidative phosphorylation, clearly extending the work of Scherrer and Monod[50] on the 'critical speed'. Ettema calculated the critical speeds (m/sec) for swimming, running, skating and cycling starting with the world records dating from 1965, obtaining values for each of these modes of 1.43, 5.85, 10.6 and 13.5 m/sec, respectively. For running and cycling these correspond to 21.06 and 48.6 km/h, respectively, which are significantly below the actual values. Ettema[49] perhaps did not sufficiently take into account the aerodynamic component of the energy cost. If this component only represents 10% of the total energy cost of running, it can be ignored. On the other hand, at 90% for cycling, it cannot. Technological progress has also meant that 49 km/h has been bettered, particularly at higher altitude where aerodynamic drag is less, as has been shown by di Prampero.[29]

The values of the coefficient a calculated by Ettema[49] for swimming, running, skating and cycling for the world records of the time are 40, 240, 180 and 200m, respectively. These values appear low because only long distances were taken into account. Ettema therefore did not question the influence of distances chosen on estimation of the value of a. Finally, Ettema concluded that the rules of fatigue elicited particularly by Kennelly[2] for the power/duration relationship in fact relate to the renewal of ATP by physiological mechanisms. That is, we can liken the muscle to a transformer of energy from its chemical form (phosphorylated bonds) to a mechanical form (locomotion).

Ten years later, Margaria et al.[51] proposed an equation predicting the time to cover a given distance by assuming the independence of the energetic cost of racing and speed between 10 and 20 km/h. They estimated that:

- the energetic cost of running is equal to 0.9 cal/m/kg[52]
- the $\dot{V}O_2$ at rest (having no part in the production of energy for running) is equal to 6 ml/min/kg;
- the energy supplied by anaerobic glycolysis is equal to that supplied by the oxidative processes over 1 min (the $\dot{V}O_{2max}$), and therefore an athlete with a $\dot{V}O_{2max}$ of 70 ml/min/kg has an accumulated maximal oxygen deficit of 70 ml/kg.[30] A nomogram was derived from the equation:

$$d = 5(\dot{V}O_{2max} - 6)t + 5\dot{V}O_{2max} \quad \text{(Eq. 32)}$$

where d is the maximal distance (metres) covered in time t (minutes) for a given $\dot{V}O_{2max}$ (ml/min/kg). $\dot{V}O_{2max}$ can therefore be determined from the time taken for covering a given distance according to the equation:

$$\dot{V}O_{2max} = [(d + 30t)/(t + 1)]/5 \quad \text{(Eq. 33)}$$

If an athlete runs 3000m in 9 min, the calculated $\dot{V}O_{2max}$ is 65.4 ml/min/kg, which appears a little low for running 9 min at 20 km/h. Margaria et al.[52]

specified that this equation could be modified according to the runner's nutritional state, the energetic cost of the race and the individual's capacity to maintain $\dot{V}O_{2max}$ throughout the event. The notion of t_{lim} at $\dot{V}O_{2max}$ arose as a result of this article, proposing it as a condition of validity in an equation for predicting $\dot{V}O_{2max}$ assuming t_{lim} over a fixed distance of at least 5 min. If we compare the 2 conditions of validity, knowing that the duration is longer than 5 min and that the runner is capable of sustaining $\dot{V}O_{2max}$ throughout the event, we can deduce that Margaria et al.[52] assumed implicity that $\dot{V}O_{2max}$ could be sustained for less than 5 min. One of his colleagues, di Prampero, was thinking at the other extreme when he wrote that $\dot{V}O_{2max}$ could be sustained 'for about 20 min'.[53]

Gleser and Vogel[54] had already stipulated that $\dot{V}O_{2max}$ could be sustained for 10 min, 90% of $\dot{V}O_{2max}$ for 25 min, and 85% of $\dot{V}O_{2max}$ for 1 hour (which is in keeping with high-level sporting reality). Their endurance model (which they defined as being the individual capacity for accomplishing prolonged work at a given intensity) is exponential, a closer shape to reality than those which are linear or hyperbolic:

$$\log t = A \cdot L_r + B \qquad \text{(Eq. 34)}$$

where t is time in minutes, parameters A and B of the equation are described below, and L_r is the relative load or imposed work on the individual (expressed in kpm/$\dot{V}O_{2max}$). Since relative load (L_r) is a work (joules) divided by a power ($\dot{V}O_{2max}$ expressed as J/sec, because 1 ml of O_2 = 20.9J for a respiratory quotient of 0.96), then L_r is expressed in sec. The relative load is therefore a unit of time. However, Gleser and Vogel[54] would certainly have expressed L_r in kpm/min/$\dot{V}O_{2max}$ to obtain a real dimensionless estimation of the relative exercise power. We believe this to be an uncorrected error.

Although this equation of exponential form was established in an empirical way, as was that of Grosse-Lordemann and Müller,[4] it took into account:

- the onset of lactate accumulation at loads around 50 to 60% of $\dot{V}O_{2max}$
- the smaller increase in cardiac output as a result of the plateauing of the systolic ejection volume at about 60% of $\dot{V}O_{2max}$
- the depleted muscular glycogen during long races.

This equation applies to exercise intensities between 50 and 110% of $\dot{V}O_{2max}$.[54] According to these authors, it is possible to compare individuals with each other, or even with themselves following training, in terms of the parameters A (the slope of the time of endurance/load line expressed in kpm/$\dot{V}O_{2max}$ in L/min), and B which is the intercept of this line with the *x*-axis (relative load percentage $\dot{V}O_{2max}$). They showed that an increase in A was dependent on that of B, and that an increase in $\dot{V}O_{2max}$ (in L/min) alone could not affect both A and B. Indeed, if the load (kpm) increases proportionally to $\dot{V}O_{2max}$ (in L/min), their relation does not change, and endurance time is sustained because the relative load as a percentage of $\dot{V}O_{2max}$ was not altered. Intercept B could be considered as an instantaneous maximal load, and we cannot imagine an increase of both A, the individual's endurance, and B. The participants (skiers and long distance runners) followed a 10-week training programme performed at intensities of between 50 and 100% of $\dot{V}O_{2max}$, which was re-evaluated in the fifth week. Factor A only fell 0.0008 on average for the 8 participants (range –0.0071 to –0.0079). The heterogeneous unit chosen by Gleser and Vogel[54] to express the relative pedalling power in relation to $\dot{V}O_{2max}$ leads to difficulties in interpretation, because $\dot{V}O_{2max}$ was expressed in L/min and not in kpm, which would have enabled the expressed load to be seen on the abscissa as a ratio, giving load/maximal load at $\dot{V}O_{2max}$.

In the early 1980s, Moritani et al.[37] re-examined the model of Scherrer and Monod[50] concerning the relationship between W_{lim} and t_{lim} of local dynamic work and applied and validated it for general supramaximal exercise undertaken on an ergocycle:

$$W_{lim} = a + b \cdot t_{lim} \qquad \text{(Eq. 35)}$$

$$P = a/t_{lim} + b \qquad \text{(Eq. 36)}$$

Thus, for supramaximal exercise exhausting the participants between 50 sec and 4 min, a linear model describes the relationship between t_{lim} and W_{lim}.[37,38] Moritani et al.[37] stated that the critical power (equal to parameter b) was highly correlated with and close to the exercise power at the start of hyperventilation, the break-point of the ventilatory debt/exercise power relationship described by Wasserman and MacIlroy[55] as the ventilatory threshold, or anaerobic threshold. This confirms the importance of being able to maintain critical power over a prolonged period of time. Furthermore, the effects of hypoxia (the inhaling of a gas with diluted oxygen content) were studied utilising 8 volunteers in the experiment. Hypoxia reduced the critical power estimate according to the degree of hypoxia, with the value of parameter a being unchanged. This confirmed the hypothesis that parameter a represents an anaerobic energy reserve. By following the authors,[37] it was therefore possible to say:

$$W_{lim} = a + b \cdot t_{lim} = A_{max} + S_{vent} \cdot t_{lim} \qquad \text{(Eq. 37)}$$

where S_{vent} is the power relating to the ventilatory anaerobic threshold and A_{max} is the maximal anaerobic capacity.

Meanwhile, Housh et al.[56] showed that the critical power (equal to parameter b) could not be maintained beyond 30 min on average for individuals with minimal training. On the other hand, theoretically speaking, the link between critical power and the anaerobic threshold was unclear if critical power was determined from supramaximal exercises of less than 4 min. In the study by Moritani et al.,[37] the calculated critical power was equal to 80% of the maximal aerobic power.

All of the models in section 3.2, with the exeption of Wilkie,[43,44] ran counter to the calculation of $\dot{V}O_2$ for submaximal exercise power (less than $\dot{V}O_{2max}$) based on an adjustment delay which determines the inertia of the aerobic and anaerobic metabolisms (as well as their participation in ATP resynthesis).

Camus et al.[57] established an inverse relationship between supramaximal exercise intensity expressed as a percentage of $\dot{V}O_{2max}$ and exercise time (t_{lim}). Indeed, Camus et al.[57] considered that the supramaximal endurance is perhaps predicted by the intensity of exercise expressed in relation to its difference from $\dot{V}O_{2max}$ according to the equation:

$$t_{lim} = f(E - \dot{V}O_{2max}) \qquad \text{(Eq. 38)}$$

where E is the $\dot{V}O_2$ requirement of the exercise. This takes into account the difference between the energy debit of supramaximal exercise and $\dot{V}O_{2max}$, allowing the interpersonal differences of t_{lim} at these intensities to be annulled.

This function of endurance time at supramaximal exercise may be written according to the equation:

$$t_{lim} = a \cdot \exp[b(E - \dot{V}O_{2max})] \quad r = 0.979,\ p < 0.001 \qquad \text{(Eq. 39)}$$

where t_{lim} is expressed in sec, E and $\dot{V}O_{2max}$ are expressed in ml of O_2/min/kg and a and b are equal to 330.8 sec and 0.14 kg•min/ml respectively. When $E = \dot{V}O_{2max}$, $t_{lim} = 330.8$ sec. These results clearly show the significance of $\dot{V}O_{2max}$ as a criterion of physical ability in supramaximal intensity. However, $\dot{V}O_{2max}$ was not correlated to blood pH nor to lactate, which could have confirmed that $\dot{V}O_{2max}$ is in fact the difference in power upon which depends the supramaximal t_{lim} and therefore (by multiplying t_{lim} by $E - \dot{V}O_{2max}$) an accumulated oxygen deficit dependent on the capacity of the anaerobic metabolism.

3.3 Physiological Models of Increased Sophistication

A recent model describing and explaining the time/speed relationship of racing is that of Péronnet and Thibault.[58,59] As with Ettema,[49] these authors considered it possible to establish relationships between time, distance and speed from male world records for cyclic-type activities (i.e. movement repeated in an identical way at each cycle) such as running on foot, cycling, swimming and skating. Rather than being a description of a simple mathematical formulation between racing speed and distance involving several exponential functions, the model of Péronnet and Thibault attempts to explain the record for the shorter and longer distances (from 60m to a marathon) in detail from physiological considerations based on current knowledge of the maximal capacities of different metabolic systems.

This model is based on the work of Lloyd[48] and Ward-Smith.[60] In this model, the quantity of work accomplished at the moment of fatigue must equal the amount of work derived from the anaerobic and aerobic energy systems. That is to say, if A_{max} is the work equivalent of the anaerobic maximal capacity (the maximal quantity of energy that can be supplied from anaerobic metabolism), Péronnet and Thibault[58,59] assume that the total quantity of work is inferior to A_{max} for very short and very long exercise durations. Indeed, for very short exercise (<30 sec), it is not possible to use up the energy reserves of anaerobic origin. In Péronnet and Thibault's model, the anaerobic energy available at the time of short-duration supramaximal exercise (A) is given by the following formula:

$$A = A_{max}[1 - (e^{t_{lim}/k_a})] \quad \text{(Eq. 40)}$$

where k_a represents the time constant of the utilisation of these anaerobic reserves, the value of k_a being 20 sec. A is very near to A_{max} for $t_{lim} = 1$ min.

Péronnet and Thibault[59] proposed the following relationship between t_{lim} and the quantity of available anaerobic energy for exercises longer than 7 min [an estimation of the average value of t_{lim} at maximal aerobic power (MAP)]:

$$A = A_{max}[1 + f \cdot \ln(t_{lim}/t_{MAP})] \quad \text{(Eq. 41)}$$

where *f*, whose value is negative, is the decreasing rate in A with the natural logarithm of t_{lim} relative to t_{MAP} (the time for which MAP can be sustained). Therefore:

$$A = A_{max}[1 - (e^{t_{lim}/k_a})] \text{ for } t_{lim} < t_{MAP} \quad \text{(Eq. 42)}$$

$$A = A_{max}[1 + f \cdot \ln(t_{lim}/t_{MAP})] \text{ for } t_{lim} > t_{MAP} \quad \text{(Eq. 43)}$$

Concerning aerobic metabolism, this model takes account of 2 factors: (i) an inertia in aerobic metabolism exists at the start of exercise; and (ii) the value of the $\dot{V}O_2$ plateau is lower when the duration of the exhaustive exercise is prolonged. For exercises of short duration ($t_{lim} < t_{MAP}$) the value of the $\dot{V}O_2$ plateau is equal to $\dot{V}O_{2max}$ and the value of aerobic power at any instant is given by the following equation:

$$P_{aerobic} = MAP(1 - e^{t_{lim}/k_b}) \quad \text{(Eq. 44)}$$

where k_b represents the time constant of aerobic metabolism (about 30 sec). The decrease in the value of the aerobic power plateau ($P_{plateau}$) with t_{lim} for exercise exhausting participants over a time greater than t_{MAP} (i.e. $t_{lim} > t_{MAP}$) is given by the following equation:

$$P_{plateau} = MAP + E[\ln (t_{lim}/t_{MAP})] \quad \text{(Eq. 45)}$$

or:

$$P_{plateau} = MAP + E(\ln t_{lim} - \ln t_{MAP}) \quad \text{(Eq. 46)}$$

where E represents the rate of decrease in maximal oxygen consumption. The value of E (negative) is considered to be an indicator of endurance capacity.

In short, Péronnet and Thibault[59] formulated a very complete model on a sound physiological basis which allowed the world records for distances between 60m and the marathon to be predicted. For this, they took into account:

- the diminution in the quantity of energy supplied by anaerobic metabolism as the time of exercise is either prolonged or shortened
- the impossibility of maintaining MAP for more than 7 min (a duration they had not actually measured).

They also assumed that:

- the speed corresponding to MAP could be maintained for a t_{lim} of 7 min (t_{MAP})
- the power of aerobic origin available over the duration of t_{lim} ($P_{plateau}$) decreased for $t_{lim} > 7$ min in a logarithmic way, and therefore:

$$P_{plateau} = MAP + MAP \cdot S_1 \cdot \ln t_r = MAP \cdot (1 + S_1 \cdot \ln t_r) \quad \text{(Eq. 47)}$$

where S_1 is a negative constant and t_r is the value of t_{lim} expressed as a multiple of the maximal duration of the maintenance of MAP (t_{MAP}), i.e. in multiples of 7 min. For t_{lim} = 7 min (420 sec), t_r = 1, ln t_r = 0 and $P_{plateau}$ = MAP. The MAP • S_1 product is the equivalent of E, Péronnet's endurance indicator, which reflects the individual's capacity to utilise the largest fraction of $\dot{V}O_{2max}$ for the longest time possible. Better marathon runners mobilise about 85% of $\dot{V}O_{2max}$ over the 42.195km.

However, the concept of the fractional utilisation of $\dot{V}O_{2max}$ as a plateau for a certain duration of exercise does not take into account the effects of the $\dot{V}O_2$ slow component discovered by Whipp and Wasserman[61] for exercise where power is greater than the power at the anaerobic threshold. Nor does it take into account '$\dot{V}O_2$ drift', an additional component appearing after a longer time (≥30 min) of exercise at or not much below the anaerobic threshold.

Gaesser and Poole[62] reviewed the difference between these 2 additional increases in $\dot{V}O_2$ with time. The $\dot{V}O_2$ slow component is caused mainly by peripheral muscular work (86% according to Poole et al.[63]), with notably the recruitment of fast fibres (the slow component is greater for individuals with a large percentage of fast fibres). This slow component causes $\dot{V}O_2$ to increase inexorably to $\dot{V}O_{2max}$ and therefore to imminent fatigue, precipitating the cessation of exercise.[64] $\dot{V}O_2$ drift is regarded as being caused by thermoregulatory adaptations, notably an increase of skin blood flow. In any case, it seems unlikely that the true value of $\dot{V}O_2$ would be predicted at the end of exercise by using the model of Péronnet and Thibault.[59] Furthermore, t_{lim} values at MAP are somewhat variable among individuals (from 4 to 11 min[26] with a variation coefficient of 25%) although more homogenous (5%) with respect to $\dot{V}O_{2max}$.

It was by considering a slow component of the adjustment of $\dot{V}O_2$ that Hill et al.[65] proposed an alternative view of the 'critical speed' model of Monod and Scherrer[66] by considering the critical speed (CS) as being the maximal value for which the $\dot{V}O_2$ slow component did not appear. In fact he regarded t_{lim} in Monod and Scherrer's hyperbolic model (equation 26) to be the same as the time necessary to reach $\dot{V}O_{2max}$. (This still remains to be demonstrated, having been contradicted by the work of Billat and Koralsztein.[67]) Hill et al.[65] wrote:

$$P \text{ at } \dot{V}O_{2max} = a'/t_{lim} + b' \qquad \text{(Eq. 48)}$$

and:

$$W_{lim} = a' + b' \cdot t_{lim} \qquad \text{(Eq. 49)}$$

This equation is validated for exercise intensities between 95 and 110% of MAP. b′ is the critical power (corresponding to CS′), the maximal value for which the $\dot{V}O_2$ slow component does not appear. Hill et al.[65] did not demonstrate a significant difference between CS and CS′. They then concluded that critical power (classically defined as the total time sustained at a given power and not the delay in reaching $\dot{V}O_{2max}$) was reached above the start of the lactate accumulation threshold, and above the maximal speed for which we are still certain of reaching a lower plateau to that of $\dot{V}O_{2max}$, and therefore of not being prematurely fatigued. This critique could be a recognition of possible causes for volitional stopping of exercise before reaching $\dot{V}O_{2max}$.

di Prampero,[53] following a suggestion of Margaria,[68] analysed world records for different forms of locomotion using Wilkie's[44] equation (equation 27) which he reformulated as:

$$E_{max} = AnS \cdot t^{-1} = MAP - MAP \cdot k^{-1}(1 - e^{-kt})t^{-1} \qquad \text{(Eq. 50)}$$

where E_{max} is the maximal metabolic power. AnS is the quantity of maximal energy supplied by anaerobic metabolism (alactic and lactic), MAP is the maximal aerobic metabolic power corresponding to $\dot{V}O_{2max}$, k is a rate constant at which $\dot{V}O_{2max}$ is reached (about 0.1 sec^{-1}); and t is the corresponding duration of exercise. E_{max} could then be calculated for world records for different forms of locomotion, comparing the records by calculating the energetic cost of locomotion. The author obtained predicted times equal to 100.2, 89.2 and 91.5% of the records of the time for running, skating and freestyle swimming, respectively.

Also following Margaria's lead, Morton[69,70] examined a hydraulic model of human bioenergetics. Margaria's original formulation[68] was solved mathematically and the solution compared with known empirical facts. It was shown that the solution conformed neither to Margaria's own graphical 'solution' nor to what actually happens under experimental conditions. These discrepancies and directions for further work are discussed by Morton,[69,70] who showed that this model, although not uniquely defined, did now conform to observed experimental data.

In a second pair of papers, Morton[71,72] examined 1 specific formulation of the 3-component hydraulic model. Various assumptions about limitations to maximal power attainment enabled Morton[72] to predict, for example, the decline in power output for continuous all-out effort such as in the Wingate test. The model prediction conforms well with published work. Furthermore, this study of maximal achievable power as a function of the existing instantaneous lactic, alactic and aerobic bioenergetic components enabled Morton to predict endurance at constant power, or rather the maximum constant power that can be maintained for a given time. This is given by an equation of form:

$$P = (a_1 + b_1e^{-r_1t} + c_1e^{-r_2t})/(a_2 + b_2e^{-r_1t} + c_2e^{-r_2t}) \quad \text{(Eq. 51)}$$

which although cutting off at 6 sec endurance where $P = P_{max}$ (the maximum 'instantaneous' power), does take a hyperbolic shape. Endurance at $\dot{V}O_{2max}$ is estimated as 9 min. Thus, Morton[72] was able to identify a 'critical' power output which in theory, could be sustained indefinitely. His estimate of this power was in the range 80 to 89% of $\dot{V}O_{2max}$, which conforms with experimentally observed data on good to elite athletes. Morton was further able to predict endurance at incremental (ramp) tests for various incremental rates. The most striking predictions are that $\dot{V}O_2$ achieved at exhaustion in an incremental test is constant, independent of increment rate (although terminal workrate is dependent) and that terminal $\dot{V}O_2$ is not equal to $\dot{V}O_{2max}$, but around 94% of it.

More recently, Morton[73] established an extension of the Monod and Scherrer[66] hyperbolic model (equation 26) which allowed the exercise power relating to t_{lim} <2 min or >30 min to be predicted more reliably. His modification of the original hyperbolic model removed the constraint according to which power needed to become infinitely large in order that $t_{lim} = 0$. Morton[73] proposed that:

$$t = a/(P - b) + k \quad \text{(Eq. 52)}$$

where k, measured in sec, is the new position of the horizontal time asymptote. Morton demonstrated that $k < 0$, which signifies that the hyperbola crosses time zero at a finite value of instantaneous maximal power, $P^* = P_{max}$. It is therefore possible to reparameterise equation 52 to include P* in the place of k, which gives a physiological meaning to all of the equation's parameters because a is the energy supplied by anaerobic metabolism, b is the critical power (close to the maximal power at steady state lactate concentration[40]) and P* is the instantaneous maximal power supplied by the ATP reserves. We therefore obtain:

$$t = a/(P - b) + a/(P - P_{max}) \quad \text{(Eq. 53)}$$

This new model formulation was tested using the experimental values obtained by McLellan and Cheung[74] using a bicycle ergometer. The estimated values of a and P_{max} are consistent with values measured from cyclists of the same standard (Péres and Vandewalle[75]). We can therefore obtain the maximal power of a cyclist according to the relationship:

$$P_{max} = 0.5\ v_0 \times 0.5\ F_0 \quad \text{(Eq. 54)}$$

where F_0 is the maximal theoretical force developed on a bicycle at speed 0 and v_0 is the theoretical speed of pedal rotation at force 0. P_{max} is the maximal power, generally between 14 and 20 W/kg bodyweight.

The critical power values of the individuals studied[74] were of the order of 250W for a value of maximal power of the order of 1619W, which is greater than that obtained by Péres and Vandewalle[75] using tests which allowed not instantaneous power but rather a sustained power for 5 to 6 sec representing full use of the alactic anaerobic metabolism. In addition, the critical power values (b) calculated with the 3-parameter model are lower than those obtained with the classic hyperbolic model.[50] Conversely, using Morton's 3-parameter model,[73] the value of parameter a was greater than that calculated with the classic 2-parameter model and conforms more to the values estimated by Wilkie.[43,44] Consequently, the value of the critical power (b) therefore approaches more closely the maximal speed at steady state lactate concentration.[76]

3.4 Significance of Endurance at $\dot{V}O_{2max}$ and its Variability

New parameters can be integrated in the modelling of the intensity/duration relationship to explain the variability in t_{lim} at the same relative exercise power, for example at 100% of $\dot{V}O_{2max}$. Although the speed/time relationship has been widely investigated for almost a century, we have not concerned ourselves until now with endurance at $\dot{V}O_{2max}$, which is integrated into most of the models describing the loss of speed with time when the $\dot{V}O_2$ adjustment delay has been accurately calculated.

The concept of $\dot{V}O_{2max}$ is an essential factor when modelling the capacities of the cardiorespiratory and muscular systems to extract, transport and use oxygen in order for oxidative phosphorylation to supply energy needs at $v\dot{V}O_{2max}$ (the running speed that just elicits $\dot{V}O_{2max}$). However, since the oxygen deficit permits faster running during short periods of time in order for the difference between the required exercise power and $\dot{V}O_{2max}$ to be met, we have investigated the possibility of explaining the duration of exercise at $v\dot{V}O_{2max}$ by the oxygen deficit, which may be more variable in size than $\dot{V}O_{2max}$ in long-distance runners.[77,78] Our studies show an inverse relationship between $\dot{V}O_{2max}$ and t_{lim} at $\dot{V}O_{2max}$.[76-78] This inverse relationship can be explained by the model of Monod and Scherrer,[66] which provides a new interpretation beyond the simple power/duration relationship. Indeed, what is predicted for an individual, a reduction in sustained power with lengthening of exercise duration, also applies to a population of elite runners, whose $\dot{V}O_{2max}$ maintenance time is relatively shorter than the extent to which their $\dot{V}O_{2max}$ is elevated. This inverse relationship between $\dot{V}O_{2max}$ and t_{lim} at $\dot{V}O_{2max}$ among 38 long distance runners is given by $\dot{V}O_{2max} = 71.4 + 5.5$ ml/kg/min and $v\dot{V}O_{2max} = 21.8 + 1.2$ km/h, $r = -0.347$, $p < 0.05$.[76] They all ran at 100% of their $\dot{V}O_{2max}$, but at absolute $\dot{V}O_{2max}$ values varying from 60 to 88 ml/min/kg. These experimental values match those of Monod and Scherrer.[66] In this mode, where P is the power that a runner can sustain until exhaustion, a W_{lim} can be calculated for the t_{lim} associated with P:

$$P = W_{lim}/t_{lim} \qquad \text{(Eq. 55)}$$

In this model, b is the critical power that can be calculated from the slope of the relationship between the racing distance accomplished for a time of exhaustion (t_{lim} in sec) and the intercept on the ordinate axis, a is assumed to be the anaerobic maximal capacity, the quantity of energy supplied by the anaerobic metabolism. We know from equation 26 that:

$$t_{lim} = a/(P - b) \qquad \text{(Eq. 56)}$$

If P is the racing speed associated with $\dot{V}O_{2max}$, i.e. $v\dot{V}O_{2max}$, then t_{lim} is the time limit at $v\dot{V}O_{2max}$. Also, b can be estimated from the maximal fraction of $\dot{V}O_{2max}$ at the lactate threshold, since it was shown by Lechevalier et al.[40] that the critical power (as a percentage of the velocity associated with $\dot{V}O_{2max}$) is not significantly different from the lactate threshold. Given that $b = Fv\dot{V}O_{2max}$ (the fraction of the velocity associated with $\dot{V}O_{2max}$ at the lactate threshold) Lechevalier et al.[40] concluded that:

$$t_{lim} = a/(v\dot{V}O_{2max} - Fv\dot{V}O_{2max}) \qquad \text{(Eq. 57)}$$

In this model, the highest t_{lim} value at $v\dot{V}O_{2max}$ is obtained with a high value of a and a low value of $(P - b)$, which is the difference between $v\dot{V}O_{2max}$ and the critical power (speed). In our experimental approach, as with the model of Monod and Scherrer[66] (equation 26), the runners with the longest t_{lim} at $v\dot{V}O_{2max}$ are those who have the smallest difference between $v\dot{V}O_{2max}$ and the fraction of $v\dot{V}O_{2max}$ at the lactate threshold and hence the smallest $v\dot{V}O_{2max}$ value (km/h).

Further research will examine the relationship between exhaustion time at $\dot{V}O_{2max}$ and the anaerobic capacity expressed as quantity of energy (joules). Other questions to be asked are whether runners with a high $\dot{V}O_{2max}$ also have shorter t_{lim} at $\dot{V}O_{2max}$ for a similar $\dot{V}O_{2max}$ spread, and whether such differences are seen in sportsmen of the same aerobic ability undertaking events necessitating different muscle groups, such as swimming, cycling, canoeing and running).[79]

4. Conclusions

A study of the relationship linking exercise power with endurance by referring to world records, generally for running, has allowed physiologists to better characterise the 3 types of energy metabolism in humans. It has even been claimed that some characteristics of the intensity/duration relationship provide an index for an athlete's endurance, such as the capacity to sustain the highest fraction of maximal power for the longest time possible. This maximal power can be the aerobic maximal power (minimal power which solicits $\dot{V}O_{2max}$) or even the absolute maximal power taken over very short durations (a few seconds).

These considerations have led physiologists to propose theoretical models based on human energetic characteristics, i.e. the power, maximal capacities and inertia of the aerobic and anaerobic metabolic systems. Endurance can therefore be characterised as the slope of the power (or $\dot{V}O_2$ as a fraction of $\dot{V}O_{2max}$) and duration relationship. The interpersonal variability of t_{lim} at $\dot{V}O_{2max}$, and the inverse relationship between $\dot{V}O_{2max}$ and t_{lim} at $\dot{V}O_{2max}$, show how the oxygen deficit determines the general form of this intensity/duration relationship curve for exercise power levels lying between the onset of lactate accumulation and $\dot{V}O_{2max}$.[67] Also, the slow adjustment of $\dot{V}O_2$ ('$\dot{V}O_2$ slow component') which appears in this intensity domain could explain the variability in t_{lim} at these power levels, causing individuals to perform at greater fractions of $\dot{V}O_{2max}$. However, the link between the delay in reaching $\dot{V}O_{2max}$ for exercise intensities lying between the lactate threshold and the maximal aerobic power and the delay in appearance of fatigue has not yet been established.

Finally, the study of this intensity/duration relationship allows us to establish the athlete's energetic profile and advise individuals in their choice of competition distance, as long as the points that represent the record times for particular distances are situated above or below the individual's average curve. The prediction of human records over distances that are not the subject of competition at present seems possible with the model of Péronnet and Thibault,[58] which currently remains the most effective from the point of view of the combination of metabolism and sporting performances. Furthermore, these authors offered a way of assessing human endurance according to the slope of the relationship between the fractional utilisation of $\dot{V}O_{2max}$ and exercise duration (as a natural logarithm). However, this model assumes an invariant t_{lim} at $\dot{V}O_{2max}$ (7 min) in providing a system of reference in the calculations of the endurance index. This is why we suggest caution in the use of this endurance index.

The most recent 3-component model by Morton,[73] which extends the judicious historical model of Monod and Scherrer,[66] allows for the instantaneous maximal power as well as anaerobic energy and critical power (which is close to the power at the lactate threshold) to be calculated.

References

1. Kennelly AE. An approximate law of fatigue in the speeds of racing animals. Proc Am Acad Arts Sci 1906; 42 (15): 275-331
2. Kennelly AE. Changes during the last twenty years in the world's speed records for racing animals. Proc Am Acad Arts Sci 1926; 61 (11): 486-523
3. Meade GP. An analytical study of athletic records. The Scientific Monthly 1916; 2: 596-600
4. Grosse-Lordemann H, Müller EA. Der einfluss der leistung und der arbeitgeschwindigkeit auf das arbeitsmaximum und den workungsgrad beim radfahren. Arbeitsphysiol 1937; 9: 454-75

5. Tornvall G. Assessment of physical capabilities. Acta Physiol Scand 1963; 58 Suppl.: 201
6. Purdy JG. Least squares model for the running curve. Res Q 1974; 45: 224-38
7. Francis AW. Running records. Science 1943; 93: 315-6
8. Edwards RHT, Toescu V, Gibson H. Historical perspective: a framework for interpreting pathobiological ideas on human muscle fatigue. In: Simon C, Gandevia H, editors. Fatigue. New York: Plenum Press, 1995: 481-94
9. Henry FM, Farmer DS. Condition ratings and endurance measures. Res Q 1949; 20: 126-33
10. Henry FM. Time-velocity equations and oxygen requirements of 'all-out' and 'steady-pace' running. Res Q 1954; 25: 164-77
11. Lietzke MH. An analytical study of world and Olympic racing records. Science 1954; 119: 333-6
12. Lietzke MH. Consistent set of running times. Science 1956; 124: 178
13. Meade GP. Consistent running records. Science 1956; 124: 1025
14. Péronnet F. Les records du monde de course à pied masculins et féminins: à propos d'un article de la revue 'Nature'. Rev STAPS 1993; 32: 47-55
15. Whipp BJ, Ward SA. Will women soon outrun men [letter]? Nature 1992; 355: 25
16. Henry FM. Prediction of world records in running sixty yards to twenty-six miles. Res Q 1955; 26: 147-58
17. Craig AB. Evaluation and predictions of world running and swimming records. J Sports Med Phys Fitness 1963; 3: 14-21
18. Keller JB. A theory of competitive running. Physics Today 1973; 26: 42-7
19. Ryder HW, Carr HJ, Herget P. Future performance in footracing. Sci Am 1976; 234: 108-19
20. Frederick EC. A statistical model of endurance in running. Can J Appl Sport Sci 1977; 2: 127-32
21. Coleman CE, Rumball WM. The fatigue coefficient. Track Technol 1973; 53: 1694-5
22. Morton RH. The supreme runner: what evidence now? Aust J Sports Sci 1983; 3: 7-10
23. Morton RH. The supreme runner: a theory of running and some of his physiological attributes. Aust J Sci Med Sport 1984; 16: 26-8
24. Noakes TD. Lore of running. 3rd ed. Champaign (IL): Leisure Press, 1991
25. Harman EA, Knuttgen HG, Frykman PN, et al. Exercise endurance time as function of percent maximal power production. Med Sci Sports Exerc 1987; 19: 480-5
26. Billat V, Pinoteau J, Petit B, et al. Reproducibility of running time to exhaustion at $\dot{V}O_{2max}$ in sub-elite runners. Med Sci Sports Exerc 1994; 26: 254-7
27. Kachouri M, Vandewalle H, Huet M, et al. Is the time at maximal aerobic speed an index of aerobic endurance? Arch Physiol Biochem 1996; 104: 330-6
28. Hill AV. Muscular movement in man. New York: McGraw-Hill, 1927: 104
29. di Prampero PE. The energy cost of human locomotion on land and in water. Int J Sports Med 1986; 7: 55-72
30. Medbo JI, Mohn AC, Tabata I, et al. Anaerobic capacity determined by maximal accumulated O_2 deficit. J Appl Physiol 1988; 64: 50-60
31. Daniels J, Scardina N, Hayes J, et al. Elite and subelite female middle- and long-distance runners. In: Landers D, editor. Sport and elite performers. Champaign (IL): Human Kinetics, 1984: 57-72
32. Sargent RM. The relation between oxygen requirement and speed in running. Proc R Soc Lond Ser B 1926; 100: 10-22
33. Léger L, Boucher R. An indirect continuous running multistage field test, the université de Montréal Track Test. Can J Appl Sport Sci 1980; 5: 77-84
34. Margaria R, Edwards HT, Dill DB. The possible mechanism of contracting and paying the oxygen debt and the role of lactic acid in muscular contraction. Am J Physiol 1933; 106: 689-719
35. Scherrer J, Samson M, Paleologue A. Etude du travail musculaire et de la fatigue. J Physiol (Paris) 1954; 46: 887-916
36. Monod H. Contribution à l'étude du travail statique. Paris: Thèse de Médecine, 1956: 121
37. Moritani T, Nagata A, De Vries HA, et al. Critical power as a measure of physical working capacity and anaerobic threshold. Ergonomics 1981; 24: 339-50
38. Vandewalle H, Kapitaniak B, Grun S, et al. Comparison between a 30-s all-out test and a time-work test on a cycle ergometer. Eur J Appl Physiol 1989; 58: 375-80
39. Hughson RL, Orok CJ, Staudt LE. A high velocity running test to assess endurance running potential. Int J Sports Med 1984; 5: 23-5
40. Lechevalier JM, Vandewalle H, Chatard JC, et al. Relationship between the 4 mMol running velocity, the time-distance relationship and the Leger-Boucher's test. Arch Int Physiol Biochim 1989; 97: 355-60
41. Wakayoshi K, Ikuta K, Yoshida T, et al. Determination and validity of critical velocity as an index of swimming performance in the competitive swimmer. Eur J Appl Physiol 1992; 64: 153-7
42. Wakayoshi K, Yoshida T, Udo M, et al. A simple method for determining critical speed as swimming fatigue threshold in competitive swimming. Int J Sports Med 1992; 13: 367-71
43. Wilkie DR. Man as a source of mechanical power. Ergonomics 1960; 3: 1-8
44. Wilkie DR. Equations describing power input by humans as a function of duration of exercise. In: Cerretelli P, Whipp BJ, editors. Exercise bioenergetics and gas exchange. North-Holland: Elsevier, 1980; 75-81
45. Margaria R, Cerretelli P, Mangili F. Balance and kinetics of anaerobic energy release during strenuous exercise in man. J Appl Physiol 1964; 19: 623-8
46. di Prampero PE, Davies TM, Cerretelli P, et al. An analysis of O_2 debt contracted in submaximal exercise. J Appl Physiol 1970; 29: 547-51
47. Frederick WSF. Physiological aspects of human fatigue. Arch Industr Hlth 1959; 20: 297-302
48. Lloyd BB. The energetics of running: an analysis of world records. Adv Sci 1966; 22: 515-30
49. Ettema JH. Limits of human performance and energy-production. Int Z Angew Physiol Einschl Arbeitsphysiol 1966; 22: 45-54
50. Scherrer J, Monod H. Le travail musculaire local et la fatigue chez l'homme. J Physiol (Paris) 1960; 52: 419-501
51. Margaria R, Aghemo P, Pinera Limas F. A simple relation between performance in running and maximal aerobic power. J Appl Physiol 1975; 38: 351-2
52. Margaria R, Cerretelli P, Aghemo P, et al. Energy cost of running. J Appl Physiol 1963; 18: 367-70

53. di Prampero PE. Energetics of world records in human locomotion. In: Wieser K, Gaigner L, editors. Energy transformations in cells and organisms. Amsterdam: Wieser et Gnaiger Elsevier, 1989: 248-53
54. Gleser MA, Vogel JA. Endurance capacity for prolonged exercise on the bicycle ergometer. J Appl Physiol 1973; 34: 438-42
55. Wasserman K, Mac Ilroy MB. Detecting the threshold of anaerobic metabolism in cardiac patients during exercise. Am J Cardiol 1964; 14: 844-52
56. Housh DJ, Housh TJ, Bauge SM. The accuracy of critical power test for predicting time to exhaustion during cycle ergometry. Ergonomics 1989; 32: 997-1004
57. Camus G, Juchmès J, Thys H, et al. Relation entre le temps limite et la consommation maximale d'oxygène dans la course supramaximale. J Physiol (Paris) 1988; 83: 26-31
58. Péronnet F, Thibault G. Analyse physiologique de la performance en course à pied, révision du modèle hyperbolique. J Physiol (Paris) 1987; 82: 52-60
59. Péronnet F, Thibault G. Mathematical analysis of running performance and world running records. J Appl Physiol 1989; 67: 453-65
60. Ward-Smith AJ. A mathematical theory of running, based on the first law of thermodynamics and its application to the performance of world-class athletes. J Biomech 1985; 18: 337-49
61. Whipp BJ, Wasserman K. Oxygen uptake kinetics for various intensities of constant-load work. J Appl Physiol 1972; 5: 351-6
62. Gaesser GA, Poole DC. The slow component of oxygen uptake kinetics in humans. Exerc Sport Sci Rev 1996; 24: 35-70
63. Poole DC, Schaffartzik W, Knight DR, et al. Contribution of exercising legs to the slow component of oxygen uptake kinetics in humans. J Appl Physiol 1991; 71: 1245-53
64. Whipp BJ. The slow component of O_2 uptake kinetics during heavy exercise. Med Sci Sports Exerc 1994; 26: 1319-26
65. Hill D, Billat V, Williams CS, et al. The relationship between running velocity and the time to achieve $\dot{V}O_{2max}$. J Appl Physiol. In press
66. Monod H, Sherrer J. The work capacity of a synergic muscular group. Ergonomics 1965; 8: 329-38
67. Billat V, Koralsztein JP. Significance of the velocity at $\dot{V}O_{2max}$ and its time to exhaustion at this velocity. Sports Med 1996; 22: 90-108
68. Margaria R. Biomechanics and energetics of muscular exercise. Oxford: Oxford University Press, 1976
69. Morton RH. On a model of human bioenergetics. Eur J Appl Physiol 1985; 54: 285-90
70. Morton RH. A three component model of human bioenergetics. J Math Biol 1986; 24: 451-66
71. Morton RH. On a model of human bioenergetics: II. Maximal power and endurance. Eur J Appl Physiol 1986; 55: 413-8
72. Morton RH. Modelling human power and endurance. J Math Biol 1990; 28: 49-64
73. Morton RH. A 3-parameter critical power model. Ergonomics 1996; 39: 611-9
74. McLellan TM, Cheung SY. A comparative evaluation of the individual anaerobic threshold and the critical power. Med Sci Sports Exerc 1992; 24: 543-50
75. Péres G, Vandewalle H, Monod H. Aspect particulier de la relation charge vitesse lors du pedalage sur cycloergometre. J Physiol (Paris) 1987; 82: 52-60
76. Billat V, Pinoteau J, Petit B, et al. Time to exhaustion at 100% of velocity at $\dot{V}O_{2max}$ and modelling of the time limit/velocity relationship in elite long distance runners. Eur J Appl Physiol 1994; 69: 271-3
77. Billat G, Pinoteau J, Petit B, et al. Speed and time to exhaustion at $\dot{V}O_{2max}$ contribute to long distance performance in sub-elite runners. Science et Motricité 1995; 27: 40-50
78. Billat V, Beillot J, Rochcongar P, et al. Gender effect on the relationship of time limit at 100% $\dot{V}O_{2max}$ with other bioenergetics characteristics. Med Sci Sports Exerc 1996; 28: 1049-55
79. Billat V, Faina M, Sardella F, et al. A comparison of time to exhaustion at $\dot{V}O_{2max}$ in elite cyclists, kayak paddlers, swimmers and runners. Ergonomics 1996; 39: 267-77

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