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ORIGINAL ARTICLE

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The critical power model for intermittent exercise

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Abstract This paper develops and illustrates the critical power model for intermittent work. Model theoretic development reveals that total endurance time is always a step function of one or more of the four independent variables: work interval power output (P_w), rest interval power output (P_r), work interval duration (t_w), and rest interval duration (t_r). Six endurance-trained male athletes recorded their best performances during the season in 3-, 5-, and 10-km races, and performed three different intermittent running tests to exhaustion in random order, recording their total endurance times. These data were used to illustrate the model and compare anaerobic distance capacities (α) and critical velocities (β) estimated from each type of exercise. Good fits of the model to data were obtained in all cases: $0.954 < R^2 < 0.999$. Critical velocity was found to be significantly less when estimated using an intermittent versus continuous running protocol.

Keywords Endurance · Exhaustion · Fatigue · Hyperbolic model · Running

Introduction

The critical power (CP) concept (Moritani et al. 1981 Hill 1993) provides a simplified two-component (anaerobic and aerobic) model of the human bioenergetic system. During exercise, depletion of the anaerobic work capacity (α, measured in joules) is supplemented by an aerobic supply of maximum rate (β, measured in watts). These are two important parameters which characterise human work performance. Anaerobic work capacity is that parameter representing the aggregate total work that can be performed by the body's limited fuel reserves (phosphagens, glycolysis resulting in net lactate production, and oxygen stores), regardless of the rate at which these reserves are utilised. The maximum aerobic supply rate is that parameter representing the upper limit for sustainable power, a power that could, in theory, allow work for an infinite length of time. For this reason it is commonly referred to as the 'critical power'.

Application of the CP concept to exercising humans has been predominantly for continuous constant power protocols, typically in cycling or in running (where α is measured in metres and β in metres per second). The study of endurance ability has been the main focus (Morton and Hodgson 1996; Billat et al. 1999). More recently, the CP concept has been applied to continuous exercise of a ramp protocol (Morton 1994, 1997). Estimates of the values α and β from constant power and from ramp protocols by the same subjects have not been found significantly different (Morton et al. 1997).

During intermittent exercise (Christensen 1960), intervals of work and rest (or relative rest) are performed alternately. Such a protocol is popular in conditioning programmes and is known as interval training. Here, various work-to-rest time intervals and/or powers can be employed as a means of training the various systems of energy transfer. In so doing, much lower blood lactate thresholds and a far greater capability for exercise have been observed compared to a continuous protocol (Margaria et al. 1969; Astrand and Rodahl 1970). As far as can be determined, the CP concept has never been applied to intermittent exercise. This may be somewhat unexpected, since a similar bioenergetic model theory incorporating recovery after exercise has already been suggested (Morton 1986).

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The purpose of this paper is to present the CP model theory adapted for intermittent exercise (illustrated with running data collected from several subjects) and to examine whether the anaerobic (distance) capacity and the critical velocity in the same subjects differ, depending on whether estimated from continuous or intermittent running.

Model development

Theory

When the CP concept is applied to endurance at continuous constant power exercise, the most natural dependent variable is the total endurance time, t (s). Anaerobic work capacity and critical power are the two parameters of the system and power output is the single independent variable. When applied to intermittent exercise, there are four independent variables to consider: the power (W) during the work and rest phases, P_w and P_r , respectively; and the length of the time (s) intervals of the work and rest phases, t_w and t_r, respectively.

Application of the CP concept during the work intervals follows standard practice (Moritani et al. 1981; Hill 1993), though it is recognised that this is not exact (Wilkie 1980; Morton 1996; di Prampero 1999). Thus, transitions from rest to work (and vice versa), both in terms of power output (i.e. speed of running) and from the bioenergetic point of view, are regarded both as instantaneous and power- (or speed-) independent. Also, during the rest intervals it is assumed that the aerobic supply continues at the constant rate β, at least for the duration of the interval. Taking some or all of these issues into consideration, such as for example by Wilkie (1980), di Prampero et al. (1993), or Morton (1996), would necessarily complicate the model. This alternating cycle of a drain on the anaerobic capacity during work, followed by a partial refilling during rest, continues repeatedly until the anaerobic capacity is fully depleted. The time at which this occurs is the total endurance time.

Although it may appear at first sight that all the four independent variables (P_w, P_r, t_w, and t_r) can be separately manipulated, there are some restrictions on permissible individual, or combinations of values. If these restrictions are not satisfied, the concept cannot meaningfully be applied to endurance for intermittent exercise. These restrictions are:

$$0 \leqslant P_r < \beta < P_w < \beta + \alpha/t_w \tag{1}$$

These inequalities are necessary, respectively, for the following reasons. Although P_r is often zero, it need not necessarily be so. However, it must be less than the critical power, otherwise no recovery occurs during the rest intervals. (We recognise that under the most general conditions, this need not be so, in which case $P_r < P_w$ only). P_w must exceed the critical power in order for any of the anaerobic capacity to be utilised during the work intervals. If this were not the case, endurance would be infinite (in theory). However, P_w must not be so high that exhaustion occurs before the end of the first work interval. If this were to happen, the notion of intermittent exercise would be vacuous.

$$(\beta - P_r)t_r < (P_w - \beta)t_w \tag{2}$$

or equivalently

$$(P_w t_w + P_r t_r)/(t_w + t_r) > \beta \tag{3}$$

This ensures that the partial refilling of the anaerobic capacity during each rest interval is less than the drain on capacity during each immediately preceding work interval. If this were not the case, endurance would be infinite (in theory). Alternately, the average power output over the work and rest cycle must be greater than the critical power, otherwise endurance would again become infinite. Yet another way of interpreting this restriction is to note that exhaustion cannot occur during a short period at the start of each work interval. This is due to the partial refilling, of an amount $(\beta - P_r)t_r$ J, which has occurred in the anaerobic capacity during the immediately preceding rest interval. This short period of time is of length $(\beta - P_r)t_r / (P_w - \beta)$ s. Obviously, exhaustion also cannot occur during any of the rest intervals, since $P_r < \beta$.

For example, consider a bout of intermittent exercise as described above. Suppose $\alpha = 20{,}000$ J, $\beta = 200$ W, $P_w = 400$ W, $P_r = 100$W, $t_w = 30$ s, and $t_r = 20$ s. The power output at any instant until exhaustion can be represented by Fig. 1a.

The drain on anaerobic capacity during each work interval is at the rate $(P_w - \beta)$ W, amounting in each complete work interval to $(P_w - \beta)t_w$ J. During each rest interval however, anaerobic capacity is partially refilled at the rate $(\beta - P_r)$, amounting in each complete rest interval to $(\beta - P_r)t_r$ J. The anaerobic capacity at any instant can then correspondingly be represented by Fig. 1b.

Suppose, as we have argued above, that an observed t starting with a work interval consists of a whole number (n) of complete (work + rest) cycles of duration $(t_w + t_r)$ each, plus a further partial work interval which terminates in exhaustion.

During these n complete cycles, the total drain on anaerobic capacity has been $n(P_w - \beta)t_w$, while the total refilling amount has been $n(\beta - P_r)t_r$. The anaerobic capacity that remains at the start of the final partial work interval is therefore an amount:

$$\alpha - n[(P_w - \beta)t_w - (\beta - P_r)t_r] \tag{4}$$

which amount drains thereafter at the rate $P_w - \beta$.

The total endurance time therefore is given by:

$$t = n(t_w + t_r) + \frac{\alpha - n[(P_w - \beta)t_w - (\beta - P_r)t_r]}{P_w - \beta} \tag{5}$$

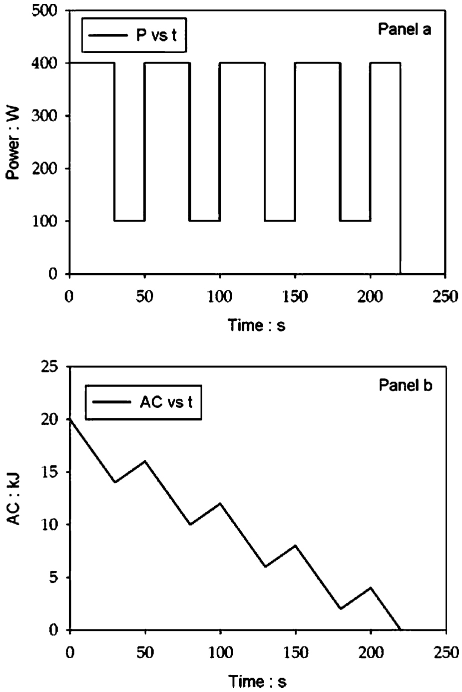


Fig. 1 Power output (P) and anaerobic capacity (*AC*) during intermittent exercise. **a** The power output at any time (t) during an intermittent exercise bout, either during a work or a rest interval, up until exhaustion at t = 220 s. **b** How anaerobic capacity drains and partially refills during the work and rest intervals respectively, up until exhaustion

Data for any one subject over a set of differing exercises may be obtained by selecting suitable values for P_w, t_w, P_r, and t_r, then counting n and measuring t. Subject specific estimates of α and β may be obtained by fitting Eq. 5 to each data set.

Illustration

Following Fig. 1, consider the example subject above with $\alpha = 20{,}000$ J and $\beta = 200$ W. Consider intermittent values of $P_w = 400$ W, $P_r = 100$ W, $t_w = 30$ s, and $t_r = 20$ s. If we fix any three at these values and let the fourth vary over its range permitted by the restrictions given above, then we obtain four versions of Eq. 5 illustrated, respectively, in the panels of Fig. 2.

In all cases, the hyperbolic nature of the CP model is evident, as are the discrete steps in the curves characteristic of the intermittent nature of the exercise. All curves slope step-wise in the direction intuitively expected.

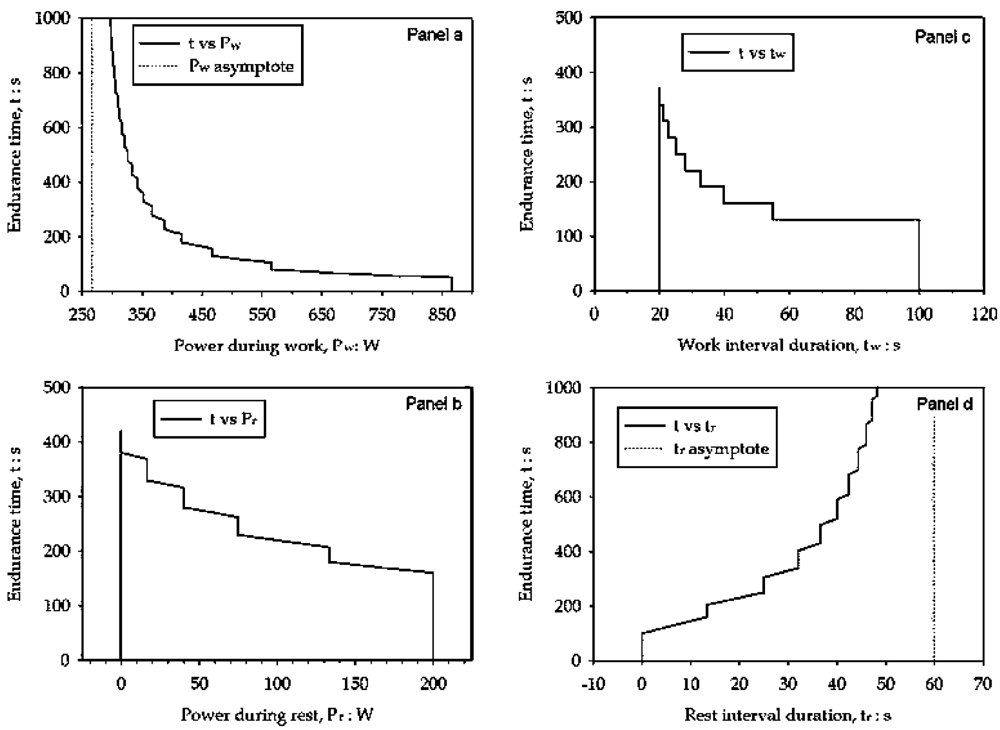
Figure 2c (t vs t_w) is curious in that the line segments between steps are flat, indicating that between each jump, when n is constant, t is also constant independent of t_w in that range. We are not aware of any published data reporting this observation. This can be proved true for any feasible combination of values of P_w, P_r, and t_r, as t_w varies within the range between each jump (see Appendix).

Methods

Subjects

Six endurance-trained male athletes [mean (SD): age 51 (6) years, height 175.0 (5) cm, and weight 71 (4) kg] volunteered to participate in this study. They trained four times per week [mean running distance (SD) 65 (18) km/week] with continuous running, but were not familiar with either severe intermittent or continuous running. Prior to participation, all subjects had a preliminary cardiological assessment with an exhaustive test on a cycle ergometer. All were

Fig. 2 Illustrative endurance times as functions of P_w work interval power output (**a**), P_r rest interval power output (**b**), t_w work interval duration (**c**) and t_r rest interval duration (**d**), respectively. In all cases, the discrete jumps are vertical. In **a** only, the line segments have slight curvature; all others are straight



assessed fit to participate. The experimental procedures complied with all ethical requirements of Universite Lille, where they took place.

Each subject's critical velocity for continuous running was calculated from their best performances during the season in 3-, 5-, and 10-km races. This critical velocity was calculated by fitting the equation of Ettema (1966):

$$D_{\mathrm{lim}} = \alpha + \beta * t_{\mathrm{lim}} \quad (6)$$

using an ordinary linear least squares method, where α is considered to be the anaerobic distance running capacity, and the slope β is termed the critical velocity (Hughson et al. 1984).

Experimental design

Subjects performed three intermittent running tests in random order. They did only one test on any given day and tests were each separated by ≥48 h and completed within the period of a week. All tests were performed on a synthetic 400-m track at the same time of day (between 1000 hours and 1600 hours) in a climate of 19–22°C without wind. Subjects were permitted to have easy jogging of 40 min only on the day(s) separating any two tests.

They were asked to refrain from food or beverages containing caffeine before testing. Runners followed a pacing cyclist travelling at the required velocity. The cyclist received audio cues via a Walkman, the cue rhythm determining the speed needed to cover 20 m. Visual marks were set at 20-m intervals along the track (inside the first lane).

These three tests consisted of alternating fast and slow running intervals until exhaustion. Running velocities were selected based on the critical velocity calculated from continuous running as described above, and interval durations were selected to provide a range of exercises. The three combinations performed by each subject were: (1) 60 s fast running at 120% of critical velocity, followed by 60 s slow running at 50% of critical velocity; (2) 180 s fast running at 110% of critical velocity, followed by 180 s slow running at 60% of critical velocity; and (3) 30 s fast running at 135% of critical velocity, followed by 60 s slow running at 65% of critical velocity.

Subjects were required to cover a set distance in each of the alternating running phases. For instance, for the intermittent running bout3, a runner who had a critical velocity of 4 m/s was required to cover $30 \times 4 \times 1.35 = 162$ m in each 30 s performed at 135% of critical velocity, and $60 \times 4 \times 0.65 = 156$ m in each 60 s performed at 65% of critical velocity.

Subjects continued these intermittent fast and slow intervals successively until they were unable to complete the required distance in the required time, or as in several cases, declined to commence the next fast running interval. The number of complete (work + rest) cycles were counted, and the total endurance time recorded in each case.

Results

A second set of estimates of α and β were obtained for all subjects according to the fits of Eq. 5 above, using a non-linear least squares method (SigmaPlot, Jandel Scientific, San Raphael, Calif.). In all six cases good fits were obtained, with R^2 in the range 0.954 to 0.999, yielding realistic values of both α and β. Both sets of estimates are shown in Table 1.

Table 1 Anaerobic distance capacity (α) and critical velocity (β) estimated from continuous and intermittent running

Subject	Continuous		Intermittent	
Number	α (m)	β (m/s)	α (m)	β (m/s)
1	222	4.07	404	3.33
2	131	4.00	338	3.09
3	203	3.88	418	3.02
4	245	3.60	49	3.12
5	224	4.05	262	3.35
6	291	4.40	92	3.77
Mean (SD)	219 (53)	4.00 (0.26)	261 (158)	3.28 (0.27)

A paired t-test suggests that α as estimated from continuous running may be less, but not significantly so ($P = 0.31$), than when estimated from intermittent running. A larger study, with less inherent (intra-subject) variability may be able to resolve this question more assuredly. A second such test concludes that the critical velocity (β) estimated from continuous running is significantly higher than when estimated from intermittent running ($P < 0.001$). The only study we can find comparing these two parameter estimates when obtained from different modalities of the same form of exercise (Morton et al. 1997) produced equivalent estimates.

The significant difference in β estimates observed in this study means that any attempt to predict intermittent performances based on estimates derived from continuous performances is in danger of violating the restriction proved by Eq. 3. For the above six subjects, this is indeed the case in all three intermittent exercises for four of them, and in one of the three exercises, for each of the other two. More fundamentally, it may mean that the concepts embodied by critical velocity, and possibly anaerobic distance capacity as well, are physiologically different in continuous versus intermittent running. This possibility clearly merits further and deeper consideration.

Conclusion

The CP concept model can be usefully applied to continuous exercises like cycling, running, kayaking, or other forms where power output (or its proxy) can be measured. We have extended the concept to an equivalent model for intermittent running, and show that it can also be successfully applied there, yielding sensible estimates when fitted to appropriate data. A much larger comparative study utilising at least four or five exercises of each type would resolve the issue as to the equivalence or not of the estimates obtained under the different modalities.

Appendix

Consider the situation of a subject characterized by anaerobic work capacity (α) and critical power (β), exercising intermittently at work and rest powers [(P_{w}) and (P_{r}), respectively]. The rest interval duration (t_{r}) is fixed, but the work interval duration (t_{w}) is allowed to vary.

For any one of a number of sub-ranges of t_w within the overall range permitted of t_w by the restrictions of the model, suppose the subject is able to perform n complete (work + rest) cycles before becoming exhausted some time into the next work interval.

t from Equation 5 is then given by:

$$
\begin{aligned}
t &= n(t_w + t_r) + \frac{\alpha - n[(P_w - \beta)t_w - (\beta - P_r)t_r]}{P_w - \beta} \\
&= \frac{n(P_w - \beta)t_w + n(P_w - \beta)t_r + \alpha - n(P_w - \beta)t_w + n(\beta - P_r)t_r}{P_w - \beta} \\
&= \frac{\alpha + n(P_w - P_r)t_r}{P_w - \beta} \qquad (7)
\end{aligned}
$$

which does not contain t_w.

We note that if $P_w = P_r$ (or $t_r = 0$ equivalently), this reduces to the hyperbolic form of the CP model for continuous constant power:

$$t : \frac{\alpha}{P_w - \beta} \qquad (8)$$

In the case of Fig. 2c, where $\alpha = 20{,}000$ J, $\beta = 200$ W, $P_w = 400$ W, $P_r = 100$ W and $t_r = 20$ s, the equation above reduces to:

$$t = 100 + 30n \qquad (9)$$

where $n = 1, 2 \ldots 9$.

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