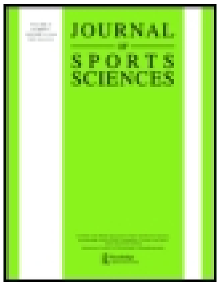

Journal of Sports Sciences
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/rjsp20

Modelling decremental ramps using 2- and 3-parameter "critical power" models

R. Hugh Morton [a] & Veronique Billat [b]

[a] Massey University, Sport and Exercise, Private Bag 11-222, Palmerston North, 4442, New Zealand

[b] Laboratoire LEPHE, Université d'Evry, Bis Impasse Christophe Colomb, Evry, 91000, France

Version of record first published: 03 Dec 2012.

Journal of Sports Sciences, 2012; 1–5, i*First* article

Routledge
Taylor & Francis Group

Modelling decremental ramps using 2- and 3-parameter "critical power" models

R. HUGH MORTON[1], & VERONIQUE BILLAT[2]

[1]*Massey University, Sport and Exercise, Private Bag 11-222, Palmerston North, 4442 New Zealand, and* [2]*Laboratoire LEPHE, Université d'Evry, 3 Bis Impasse Christophe Colomb, Evry, 91000 France*

(Accepted 1 November 2012)

Abstract

The "Critical Power" (CP) model of human bioenergetics provides a valuable way to identify both limits of tolerance to exercise and mechanisms that underpin that tolerance. It applies principally to cycling-based exercise, but with suitable adjustments for analogous units it can be applied to other exercise modalities; in particular to incremental ramp exercise. It has not yet been applied to decremental ramps which put heavy early demand on the anaerobic energy supply system. This paper details cycling-based bioenergetics of decremental ramps using 2- and 3-parameter CP models. It derives equations that, for an individual of known CP model parameters, define those combinations of starting intensity and decremental gradient which will or will not lead to exhaustion before ramping to zero; and equations that predict time to exhaustion on those decremental ramps that will. These are further detailed with suitably chosen numerical and graphical illustrations. These equations can be used for parameter estimation from collected data, or to make predictions when parameters are known.

Keywords: *anaerobic, bioenergetics, exercise testing, exhaustion, human performance modelling*

Introduction

The "Critical Power" (CP) model is a two-component supply and demand systems model of human bioenergetics, principally cycling-based. Supply is determined by the endogenous properties of the model system, and demand is determined exogenously. In its original 2-parameter form it has four basic assumptions: 1) There are only two components to the energy supply system for human exercise, termed aerobic and anaerobic; 2) The aerobic supply component is rate but not capacity limited. In cycling, this rate limiting intensity parameter is termed "critical power" and is denoted by CP, measured in watts (W); 3) The anaerobic supply component conversely is capacity but not rate limited. This capacity limiting parameter is termed "anaerobic work capacity" and is denoted by AWC, measured in joules (J); and 4) Exercise can continue at any power output $P > CP$ until AWC is exhausted.

It can easily be deduced that for any constant power, $P > CP$, the duration of exercise, t, measured in seconds (s), is given by the rectangular hyperbolic equation:

$$t = AWC/(P - CP) \tag{1}$$

In this form CP is the vertical asymptote of the hyperbola and AWC its curvature constant.

The original formulation of the CP model is attributable to Monod and Scherrer (1965) who, while studying isolated muscle exercising at fixed intensities until exhaustion, reported a linear relationship between total external mechanical work done W_k, and the time taken to do that work. Knowing that $W_k = P.t$, it can readily be noted that this linear equation must be:

$$W_k = AWC + CP.t \tag{2}$$

and that it is algebraically equivalent to the hyperbolic model equation (1) above. There are in fact six algebraically equivalent model equations linking P, t and W_k in pairs. The CP model has been extended, using appropriate rate and capacity units, to whole-

Correspondence: R. Hugh Morton, Massey University, Sport and Exercise, Private Bag 11-222, Palmerston North, 4442 New Zealand. E-mail: H.Morton@massey.ac.nz

ISSN 0264-0414 print/ISSN 1466-447X online
http://dx.doi.org/10.1080/02640414.2012.746728

body exercise in other modalities such as running and swimming, and other formats (fixed intensity, incremental ramp, and intermittent). See Hill (1993), Morton and Hodgson (1996), Billat, Koralstein, and Morton (1999), Morton (2006), and Jones, Vanhatalo, Burnley, Morton, and Poole (2010) for reviews.

The 3-parameter CP model (Morton, 1996) invokes a feedback loop which, in place of assumption 4, assumes that in cycling, the maximum voluntary external power output (MVP) at any instant, and in a range between CP and a finite maximum P_{max}, is directly proportional to the existing anaerobic capacity at that same instant. Clearly, exercise can continue as long as power demand is less than this voluntary maximum, but exhaustion (termination of exercise) occurs at the moment when the anaerobic capacity has declined to the point that the maximum voluntary power just equals the demand. It turns out that the solution to the system for constant power P produces another hyperbolic equation:

$$t = AWC/(P - CP) + k \tag{3}$$

In this formulation $k < 0$ is the horizontal asymptote of the hyperbola, and $-k$ is the MVP proportionality constant, measured in seconds. It is more intuitively appealing to physiologists to note that the intercept of the hyperbola on the horizontal axis when $t = 0$ is P_{max}, which can be interpreted as a "maximum instantaneous power". Of course for any $P > P_{max}$ no physical work is possible simply because the individual is not strong enough. This permits a reparameterisation to:

$$t = AWC/(P - CP) + AWC/(CP - P_{max}) \tag{4}$$

where $k = AWC/(CP - P_{max})$ or $P_{max} = CP - AWC/k$.

All equations apply for P constant, with $CP < P < P_{max}$. Note that when $P_{max} \to \infty$, $k \to 0$ (or vice versa) and the 3-parameter equations reduce to their 2-parameter counterparts.

The incremental ramp exercise format, in which intensity increases linearly with time from a low or more usually zero start, is of particular interest because of its widespread use in exercise-tolerance testing. The original theoretical CP model development for incremental ramps by Morton (1994) applies the 2-parameter model only. For a ramp protocol of gradient g ($W \cdot s^{-1}$, $g > 0$) from a zero start, the time to exhaustion is given by:

$$t = CP/g + \sqrt{(2AWC/g)} \tag{5}$$

The equivalent equation extended for the 3-parameter model (Jones et al., 2010) is given by:

$$t = CP/g + k + \sqrt{(k^2 + 2AWC/g)} \tag{6}$$

Conversely, decremental ramp tests necessarily starting from high intensity and with a negative gradient are used less commonly. The reason for this could simply be that little is known about them, both theoretically and empirically. Published research on decremental ramps tends to deal only with oxygen uptake kinetics (Niizeki, Takahashi, & Miyamoto, 1995; Ozyener, Rossiter, Ward, & Whipp, 2011; Yano, Yunoki, & Horiuchi, 2000). From a bioenergetic perspective, it would therefore be valuable to examine application of the CP model theory to decremental ramps. In a companion paper (manuscript in revision) we examine and compare performance and metabolic aspects of corresponding decremental and incremental ramps.

At the outset, it is important to realise that whereas incremental ramp tests will always end in volitional exhaustion of the individual, this is not necessarily so for decremental ramps. Clearly, in cycle-based exercise exhaustion occurs only if the starting power, P^*, is sufficiently high and/or the negative gradient is insufficiently steep. That is to say, in the CP model context, exhaustion of the individual will occur only if AWC is fully exhausted (2-parameter model), or if MVP declines to the intensity demanded by the ramp (3-parameter model). In practice this is quite likely to be the case.

The CP model theory and decremental ramps

The 2-parameter model

The bioenergetics of decremental ramps and determination of the boundary line separating combinations of starting intensity, $P^* > CP$, and ramp gradient, g, that will or will not lead to termination of exercise at exhaustion can be derived with reference to Figure 1.

This shows first how intensity on the ramp (the downward sloping solid line, P_r) declines from its starting value P^* with respect to time t:

$$P_r = P^* - g.t \tag{7}$$

Second, the trapezoidal area between times 0 and t, and between CP and P_r, represents the anaerobic energy requirement of the exercise up to time t. As long as this amounts to less than the anaerobic work capacity of the individual, exercise can continue, but as soon as it amounts exactly to AWC, exhaustion occurs. The time of exhaustion is therefore the point where this area equals AWC, which as indicated above, would generally occur before P_r reaches CP.

In cycling, time to exhaustion, peak power, maximal oxygen uptake ($\dot{V}O_{2max}$) and other such measures associated with the ending of an incremental ramp are frequently the main focus of that exercise modality. Given that an individual starts exercising on a decremental ramp with appropriate starting intensity and gradient; it is possible to determine endurance time as follows.

The trapezoidal area in Figure 1 is given by the expression:

$$\begin{aligned} &½[(P^* - CP) + (P^* - g.t - CP)].t \\ &= (P^* - CP).t - g.t^2/2 \end{aligned} \quad (8)$$

that when equating to AWC yields a quadratic in t, with the feasible solution given by:

$$t = (P^* - CP)/g - \left\{[(P^* - CP)/g]^2 - 2AWC/g\right\}^{1/2} \quad (9)$$

that defines time to exhaustion on an appropriately chosen decremental ramp.

If however this point occurs when P_r reaches CP exactly, exercise duration is a finite maximum and represents the boundary condition. The previously trapezoidal, and now triangular, area representing the anaerobic energy requirement of the exercise up to this time is given by the expression: $½(P^\star - CP).(P^\star - CP)/g$ or $(P^\star - CP)^2/2g$.

On equating to AWC, the equation of the boundary line is therefore:

$$g = (P^* - CP)^2/2AWC \quad (10)$$

or

$$\mathrm{P}^* = \mathrm{CP} + (2\mathrm{AWC.g})^{1/2} \quad (11)$$

This equation therefore allows determination of a critical decremental gradient for any given $P^\star$ (or vice versa) for an individual with known CP and AWC. It therefore follows that if either: $g < (P^\star - CP)^2/2AWC$ or $P^\star > CP + (2AWC.g)^{1/2}$ exhaustion of the individual will occur before reaching CP on the ramp. The converse in both cases means exhaustion will not occur and exercise will continue beyond and below CP on the ramp, even (if allowed) until ramp power reaches zero.

To illustrate, consider an individual with $CP =$ 200 W and $AWC = 25{,}000$ J exercising on a ramp starting at 400 W and decrementing at 0.25 W $\cdot s^{-1}$ (15 W $\cdot$ min^{-1}). Substitution in Equation (9) yields a time to exhaustion of 136.7 s. For the same individual, Equation (11) is given by:

$$P^* = 200 + (50000.g)^{1/2} \quad (12)$$

which is illustrated by the dotted line in Figure 2. $P^\star$ and g combinations above the line will lead to exhaustion on a decremental ramp, whereas combinations below will not. Note the location of the (400, 0.25) co-ordinate, shown by the open circle in Figure 2.

If on the other hand P_r was to reach CP before the area amounts to AWC, exhaustion does not occur, and an active recovery commences. In this case, exercise duration can be regarded as indeterminate.

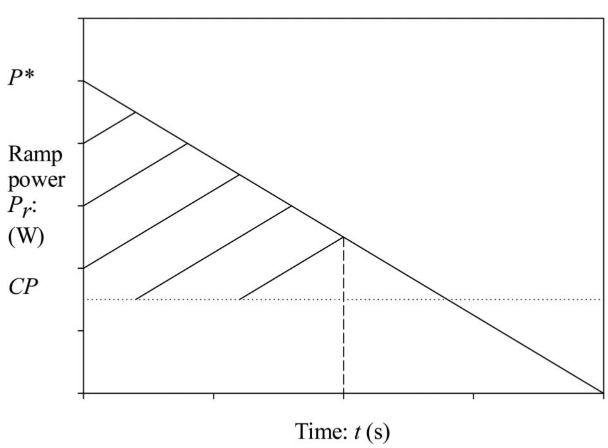

Figure 1. The downward sloping solid line, P_r, represents the power decline over time during a cycling-based decremental ramp starting at $P^\star$. The horizontal dotted line indicates the value of CP for the individual. The vertical dashed line indicates the indefinite instant in time, t. The cross-hatched trapezoidal area between times 0 and t, and between powers CP and P_r, represents the anaerobic energy requirement of the exercise up to time t.

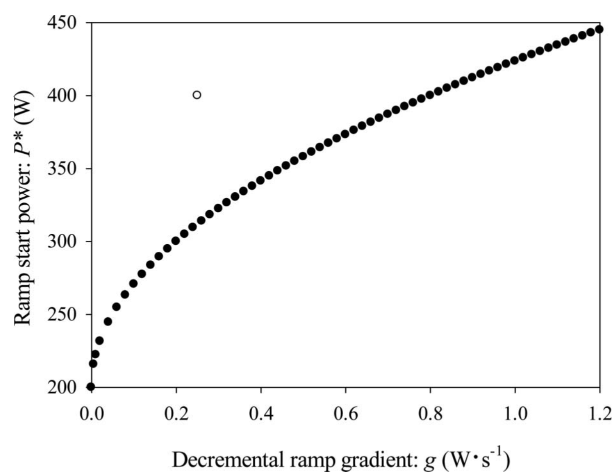

Figure 2. For an individual with $CP = 200$ W and $AWC = 25{,}000$ J (2-parameter model), the dotted line shows the boundary between those combinations of starting power on the ramp, $P^\star$, and decremental gradient, g, which will terminate in exhaustion (above and to the left); and those combinations which will not (below and to the right). The open circle is the example point (400, 0.25) which would lead to termination of exercise in 136.7 s.

It is noted in passing that Equation (11) for $P^\star$ is identical to that which defines peak power (P_{peak}) achievable on an incremental ramp of equivalent slope starting from zero watts (Morton, 2011). This must be so based on an assumption of symmetry. From a practical perspective, what this means is that CP and AWC need not necessarily be determined in an individual beforehand, though at least one incremental ramp test would be advisable before performing a decremental one.

The 3-parameter model

Given the feedback characteristic of this version of the CP concept, the modelling is a little more complex because the anaerobic capacity at any instant now needs to be considered, though we shall discover that the outcome is essentially similar. As previously, Figure 1 again applies and power on the ramp $P_r = P^\star - g.t$, but with $CP < P^\star < P_{max}$.

The anaerobic capacity (AC) of the individual starts at AWC and declines progressively over time, obtained by subtracting the corresponding trapezoidal area. That is at time t:

$$AC = AWC - (P^* - CP).t + g.t^2/2 \qquad (13)$$

The maximum voluntary power therefore will decline correspondingly over time (see Morton, 1996) from P_{max} as follows:

$$MVP = CP - AC/k \qquad (14)$$

or

$$\begin{aligned} MVP &= CP + AC.(P_{max} - CP)/AWC \\ &= CP - \left[AWC - (P^* - CP).t + g.t^2/2\right]/k \end{aligned} \qquad (15)$$

Exercise can voluntarily continue as long as $MVP > P_r$, both of which are declining over time; MVP as a quadratic starting at P_{max} as in Equation (15) and P_r linearly starting at $P^\star$ as in Equation (7), with $P^\star < P_{max}$. If these two intersect, the first point of intersection defines the instant of exhaustion and termination of exercise. The time to exhaustion can therefore be determined by simultaneous solution of Equations (7) and (15); i.e. solve for t when:

$$CP - \left[AWC - (P^* - CP).t + g.t^2/2\right]/k = P^* - g.t \qquad (16)$$

As a second illustrative example, suppose an individual has $CP = 200$ W, $AWC = 20{,}000$ J, and $k = -25$ s (i.e. $P_{max} = 1000$ W), and starts a decremental ramp of gradient $g = 1\ \mathrm{W \cdot s^{-1}}$ at $P^\star = 400$ W. Equation (16) reduces in this example to:

$$t^2 - 350t + 30{,}000 = 0 \qquad (17)$$

that has two solutions: the first and feasible is at $t = 150$ s which defines the instant of exhaustion; and the second (which is not feasible) at $t = 200$ s. This situation is depicted in Figure 3. It can be clearly seen that if the ramp decremented just a little more steeply (in fact for $g > 1.016\ \mathrm{W \cdot s^{-1}}$), exhaustion would not occur and after the initial period of high but declining intensity, a period of active recovery ensues. Mathematically speaking, Equation (16) would have no real solutions.

Continuing, and as described previously, the boundary line separating combinations of $P^\star$ and g which will, or will not, lead to exhaustion on a decremental ramp can be found when a single exact solution to Equation (16) occurs. Being a quadratic in t, this occurs when its discriminant is zero, i.e. when:

$$(k.g + P^* - CP)^2 - 4.g.[k.(P^* - CP) + AWC]/2 = 0 \qquad (18)$$

which in general has terms in $P^\star$, $P^{\star 2}$, g, g^2, $g.P^\star$, and constants. Therefore to obtain $P^\star$ given g (or vice versa) will involve solutions of this quadratic.

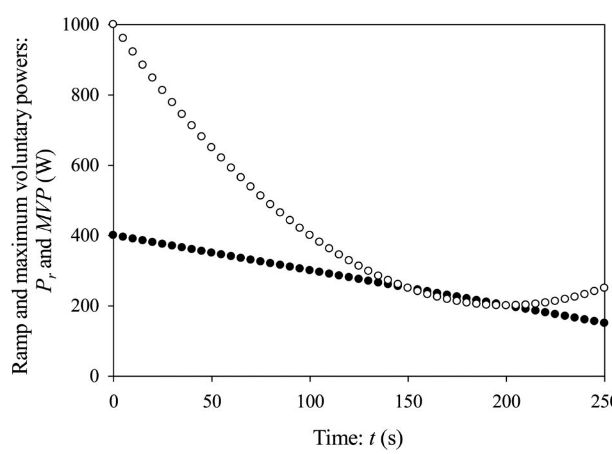

Figure 3. For an individual with $CP = 200$ W, $AWC = 20{,}000$ J, and $P_{max} = 1000$ W or $k = -25$ s (3-parameter model) on a decremental ramp of gradient $g = 1\ \mathrm{W \cdot s^{-1}}$ starting at $P^\star = 400$ W; solid circles show the declining power on the ramp, P_r, and open circles show declining MVP. The point of first intersection at $t = 150$ s defines the instant of exhaustion and termination of exercise. Continuation beyond this point shows what hypothetically would happen if the individual did not become exhausted.

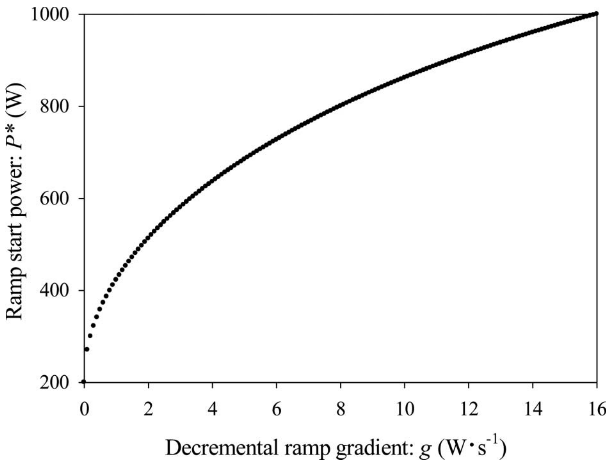

Figure 4. For the same individual in Figure 3 with $CP = 200$ W, $AWC = 20{,}000$ J, and $P_{max} = 1000$ W or $k = -25$ s (3-parameter model), the dotted line shows the boundary between those combinations of starting power on the ramp, P^*, and decremental gradient, g, which will terminate in exhaustion (above, to the left and below 1000 W); and those combinations which will not (below and to the right).

Continuing the example with the same individual, Equation (18) becomes:

$$P^{*2} - 400P^* + \left(625g^2 - 50{,}000g + 40{,}000\right) = 0 \tag{19}$$

that is illustrated by the dotted line in Figure 4.

Conclusion

For cycling, both the 2- and 3-parameter CP models can be successfully applied to derive the bioenergetic modelling of decremental ramps. Equations determining: a) the boundary between combinations of starting intensity and decremental gradient that will or will not lead to exhaustion; and b) endurance time in combinations that will lead to exhaustion, can all be determined. These equations can be used (where appropriate); either to make predictions for individuals with known *CP*, *AWC* and P_{max} values, or to estimate these values using data collected from individuals performing several decremental ramps to exhaustion.

References

Billat, V. L., Koralsztein, J. P., & Morton, R. H. (1999). Time in human endurance models. *Sports Medicine*, *27*, 359–379.

Hill, D. W. (1993). The critical power concept. *Sports Medicine*, *16*, 237–254.

Jones, A. M., Vanhatalo, A., Burnley, M., Morton, R. H., & Poole, D. C. (2010). Critical power: Implications for determination of $\dot{V}O_{2max}$ and exercise tolerance. *Medicine and Science in Sports and Exercise*, *42*, 1876–1890.

Monod, H., & Scherrer, J. (1965). The work capacity of a synergic muscular group. *Ergonomics*, *8*, 329–338.

Morton, R. H. (1994). Critical power test for ramp exercise. *European Journal of Applied Physiology*, *69*, 435–438.

Morton, R. H. (1996). A 3-parameter critical power model. *Ergonomics*, *39*, 611–619.

Morton, R. H. (2006). The critical power and related whole-body bioenergetic models. *European Journal of Applied Physiology*, *96*, 339–354.

Morton, R. H. (2011). Why peak power is higher at the end of steeper ramps: An explanation based on the "critical power" concept. *Journal of Sports Sciences*, *29*, 307–309.

Morton, R. H., & Hodgson, D. J. (1996). The relationship between power output and endurance: A brief review. *European Journal of Applied Physiology*, *73*, 491–502.

Niizeki, K., Takahashi, T., & Miyamoto, Y. (1995). A model analysis of asymmetrical response of pulmonary $\dot{V}O_2$ during incremental and decremental ramp exercise. *Journal of Applied Physiology*, *79*, 1816–1827.

Ozyener, F., Rossiter, H., Ward, S. A., & Whipp, B. J. (2011). Oxygen uptake kinetics during incremental- and decremental-ramp cycle ergometry. *Journal of Sports Science and Medicine*, *10*, 584–589.

Yano, T., Yunoki, T., & Horiuchi, M. (2000). Kinetics of oxygen uptake during decremental ramp exercise. *Journal of Sports Medicine and Physical Fitness*, *40*, 11–16.