Physica A 515 (2019) 240–247

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Detecting the marathon asymmetry with a statistical signature

Véronique Billat [a], Thomas Carbillet [b,*], Matthieu Correa [a], Jean-Renaud Pycke [c]

[a] Unité de Biologie Intégrative des Adaptations à l'Exercice, Université Evry, Université Paris-Saclay, 91025, Evry cedex, France
[b] LJK - Laboratoire Jean Kuntzmann, UJF - Université Joseph Fourier, 38401, Saint-Martin d'Hères, France
[c] Laboratoire de Mathématiques et Modélisation d'Evry, CNRS, Université Evry, Université Paris-Saclay, 91037, Evry, France

HIGHLIGHTS

- 70% of the runners tend towards having a strong asymmetric behavior.
- Runners are spending more time above their marathon average speed than below.
- The statistical signature of a marathon keeps the same order $(--)(-+)(++)(+-)$.

ARTICLE INFO

Article history:
Received 13 July 2018
Available online 4 October 2018

Keywords:
Running
Runners
Endurance
Kendall
Strava
Signature

ABSTRACT

Lately, the sub two-hour marathon attempt in Monza was still based on the belief that constant speed is the best way of running. This idea is relayed by marathon organizers who offer pace-group leaders to help the runners to maintain a target race speed. The purposes of this study are to verify the hypotheses that 1. The mass runners try to maintain a constant speed without succeeding. 2. Marathoners run in an asymmetric way and this turns out to be visible in the speed time series. Those two points are independent of the gender, the level of performance (2h30–3h40) and the profile of the race (Paris vs Berlin). Before considering a predictive running strategy for optimizing personal marathon running performance, here we shed light on some significant statistical features by analyzing speed time series data recorded by 273 runners' GPS. We started with looking for a trend in the speed time series. By means of Kendall's non-parametric rank correlation coefficient we exhibited a decreasing trend in speed data, whichever the level of performance, gender (Male and Female) and race profile (Berlin and Paris marathons). Going deeper in the study we applied a systematic analysis of the asymmetry of speed via classical statistical measures of skewness. Among them the quantiles of the average speed, i.e. the proportion of the race run above or below the final average The combination of the trend and the asymmetry lead to building up a statistical signature for the speed time series which is identical regardless the level of performance, gender and race profile.

1. Introduction

Nowadays the number of runners in the USA equals the whole French population, i.e. 65 millions with an increasing rate of 50% over the last ten years.[1] In the same way, the number of marathon finishers increased, from 353,000 finishers in the

* Correspondence to: 700 Avenue Centrale, Campus de Saint Martin d'Hères, 38401 Domaine Universitaire de Saint-Martin-d'Hères, France.
E-mail address: thomas.carbillet@billatraining.com (T. Carbillet).

[1] https://www.statista.com/statistics/227423/number-of-joggers-and-runners-usa/.

https://doi.org/10.1016/j.physa.2018.09.159
0378-4371/

Table 1
Number of marathons run divided into two categories: All Marathons (273 Marathons) and Strongly Asymmetric Speed (191 races) each subdivided in seven groups: All, Men, Women, Less than 3 h marathons, Greater-than and equal to 3 h marathons, Paris and Berlin.

	All	Men	Women	< 3 h	≥ 3 h	Paris	Berlin
Whole dataset	273	230	43	130	143	140	133
Strong asymmetry	191	162	29	91	100	97	94

early 2000s to 507,600 in 2016 [1]. In spite of an increasing volume of data split available on running community websites no study has systematically investigated a marathon running pacing strategy for the various levels of performance, from 2h30 to 3h40.

Indeed variability in pacing has been studied in respect of short and middle-distance running (e.g. 3000 m to 10 km) [2–7], these studies having especially focused on the influence of pacing on metabolic and performance measures. Ely et al. (2008) reported that elite runners completing a marathon had few changes in pace during a marathon suggesting low speed variability [8]. Even lately, the sub two-hour marathon attempt in Monza was still based on the belief that constant speed is the best way of running following the prior idea that optimal is the even pace according to the seminal model of Keller (1973) [9]. In the same way recreational marathon runners adopt the paradigm of constant speed by running with a pace-group leader from 3h00 to 4h30 every 15 min provided by the marathon organizers. Therefore the idealized paradigm of constant speed prevails in theory and perverts the practice. However, considering the physiological limitation (glycogen availability) that means that the runners have to choose the ideal speed allowing them to get the optimal performance without hitting the set of fatigue 10 km or less before the arrival ("the famous Marathon wall")!

By applying methods from mathematical statistics we test the two following hypotheses: first, most marathoners run at a speed that is significantly increasing or decreasing ; second, they run in a significantly asymmetric way leading them, for instance, to run most of the time at a speed above their average speed including the final speed fall. In addition we test a third hypothesis that these features above are independent of 1. the gender, 2. the level of performance (2h30–3h40) and 3. the profile of the race (Paris vs. Berlin) by analyzing speed time series data currently recorded by 273 runners' GPS.

More precisely, the following three points have been studied:

(1) The first one is that we notice a decreasing trend for the speed time series while running a marathon. We therefore characterize this trend statistically via Kendall's τ [10,11].

(2) Beyond the decreasing trend we point out the presence of strong positive or negative skewness in speed time series. In other words, in the case of negative asymmetry, runners spend much more time and run a longer distance above the overall marathon average speed. The combination of the trend and the asymmetry lead to building up a statistical signature for the speed time series which appears to be identical regardless the level of performance, gender and race profile.

(3) Finally we introduce a new statistical signature combining the trend and the asymmetry analyses to help runners and coaches to be able to go beyond the sole performance analysis through chronometers results. In addition before considering a predictive running strategy for optimizing personal marathon running performance, here we shed light on some significant hidden running patterns.

2. Methods

2.1. Methodology

To achieve this study we examined the time and average speed per kilometer run by 273 marathons finishers all during 2017. These data were collected from the public platform Strava.[2]

We say that a marathon has a strong positive (resp. strong negative) asymmetry if the runner spent at least 54% of the time below (resp. above) marathon average speed. More formally in a sample of speeds measured at regular intervals, a strong positive (resp. negative) asymmetry corresponds to a percentile of the average speed greater than 54% (resp. less than 46%.)

We study the marathons through different layers introducing two categories and six groups of runners. In the first category, we take the whole dataset (273 marathons), in the second one, we only take the strongly asymmetric marathons (191 marathons).

Then we split each of the two categories into seven groups: All (A), Men (M), Women (W), Less than 3 h (-3), Greater-than or equal 3 h ($+3$), Paris (P) and Berlin (B). Table 1 gives the frequencies of the categories and groups.

All races were run using a Global Positioning System (GPS) device.

2 https://www.strava.com/.

2.2. Statistical study

2.2.1. Characterizing the trend

Being aware that the accuracy of the GPS used by runners is 0.3% in open sky conditions and 1.6% in open forest or building proximity we consider the speed variable as 'continuous' even if it appears as discontinuous. Furthermore, to test the influence of the race profile, we chose two opposite marathons: Berlin having 25% of open sky conditions versus 75% of open forest/building proximity and Paris with 5% of channels, 25% of open sky and 70% of open forest/building proximity.

In order to detect the decreasing trend of speed during the marathons we are going to use the Kendall's τ [10,11]. Kendall's τ is a non-parametric coefficient computed from a times series $v_1, v_2, \ldots, v_n$. Here v_i is the ith value of a speed, $i < j$ meaning that i indicates a period of time prior to j. For instance the speed v increases with time if $v_1 < v_2 < \cdots < v_n$. Consider the symmetric kernel

$$K(v_j, v_i) = K(v_i, v_j) = \begin{cases} 1 & \text{if } i < j \text{ and } v_i < v_j, \\ 0 & \text{if } v_i = v_j, \\ -1 & \text{if } i < j \text{ and } v_i > v_j. \end{cases} \tag{1}$$

$$\tau = \frac{2}{n(n-1)} \sum_{i<j} K(v_i, v_j) \tag{2}$$

Kendall's τ is defined by Eq. (2), the sum being preformed over the $n(n-1)/2$ distinct unordered couples of indices $\{i, j\}$, so that τ takes values in between $[-1, 1]$. The Kendall's τ equals 1 (resp. -1) only if the speed time series is increasing (resp. decreasing). Furthermore a constant speed time series leads to the value $\tau = 0$. We know from the field of mathematical statistics that Kendall's is suited to detect a decreasing or increasing linear trend in a time series. It enjoys the qualities of being robust and non-parametric [12]. A p-value close to zero indicates an increasing (resp. decreasing) trend if $\tau > 0$ (resp. $\tau < 0$). Our data correspond to $n = 42$, v_i being the average speed during the ith kilometer, $1 \le i \le 42$.

2.2.2. Characterizing the asymmetry

The asymmetry of the time series $(v_i)_{1 \le i \le 42}$ will be specified in two ways:

- Time spent below vs. time spent above marathon average speed. We compute the percentage of time spent above marathon average speed for each runner.
- Comparison of the marathon median speed (v_{med}) and the marathon average speed ($\bar{v}$). We consider the sign of $\bar{v} - v_{med}$.

For the rest of this paper, we use the first coefficient (mathematically equivalent to the second one) and we consider that runners having a percentage of time spent above average speed of 54% (resp. 46%) or greater-than (resp. lesser-than) are strongly negatively (resp. positively) asymmetric. Our motivation is twofold. First this coefficient is easy to be used by every runner and coach. Second it is mathematically well-established and robust statistically.

2.2.3. Combining trend and asymmetry

Next we studied trend and asymmetry jointly. The idea was to build up a statistical signature to describe marathons speed time series. The signature is composed of two signs $(\pm, \pm)$. Thus it takes four possible values, $(++)$, $(+-)$, $(-+)$ and $(++)$. The first sign is $+$ or $-$ according to whether the Kendall τ is positive or negative. The second sign is $+$ or $-$ according to whether the asymmetry is positive or negative. In this way each runner is easily categorized. In turn, the set of signatures of one marathon, of one gender, or any other subsample can be gathered in a 2×2 matrix whose coefficients are the frequencies or relative frequencies of the four signatures in the following ordering:

$$\begin{pmatrix} -- & -+ \\ +- & ++ \end{pmatrix}$$

3. Results

3.1. Speed decreasing trend

To characterize the trend of the speed time series we use the Kendall's τ described in Section 2.2.1. Table 2 shows the results of the Kendall's τ analysis for the speed time series.

Within the 273 marathons, 81.7% of runners present a decreasing trend in speed time series with an average Kendall's τ of -0.54. The six groups (Men, Women, $<$ 3 h, $\geq$ 3 h, Paris, Berlin) highlight the same idea with the percentage of runners having a decreasing trend in the range 72.9%–90% and averages Kendall's τ close to -0.54. Looking at the p-values all less than 0.05, we can reject the null hypothesis stating that the speed time series has a constant trend.

From Table 2 we also see that 13.9% of the runners seemed to run the marathon with an increasing trend with a Kendall's τ equals to 0.30. The tendency goes into the same direction if we take a close look at the groups with the percentage of runners in the range 7.9%–20.3% and averages Kendall's τ close to 0.30.

Table 2
Kendall's τ and p-values for describing the trend in speed time series in a marathon (increasing: $\nearrow$, constant: $\longrightarrow$ or decreasing: $\searrow$) among the six groups (Men, Women, < 3 h, ≥ 3 h, Paris and Berlin).

Group	Trend	Nb. runner	% Runner	Avg. Kendall Tau	Avg. p-value
All	$\nearrow$	38	13.9	0.30	0.39
	$\longrightarrow$	12	4.4	0	< 0.05
	$\searrow$	223	81.7	−0.54	< 0.05
Men	$\nearrow$	34	14.8	0.30	0.38
	$\longrightarrow$	9	3.9	0	< 0.05
	$\searrow$	187	81.3	−0.55	< 0.05
Women	$\nearrow$	4	9.3	0.32	0.44
	$\longrightarrow$	3	7	0	< 0.05
	$\searrow$	36	83.7	−0.55	< 0.05
< 3 h	$\nearrow$	21	16.2	0.29	0.38
	$\longrightarrow$	6	4.6	0	< 0.05
	$\searrow$	103	79.2	−0.54	< 0.05
≥ 3 h	$\nearrow$	17	11.9	0.32	0.40
	$\longrightarrow$	6	4.2	0	< 0.05
	$\searrow$	120	79.2	−0.55	< 0.05
Paris	$\nearrow$	11	7.9	0.25	0.32
	$\longrightarrow$	3	2.1	0	< 0.05
	$\searrow$	126	90	−0.57	< 0.05
Berlin	$\nearrow$	27	20.3	0.32	0.42
	$\longrightarrow$	9	6.8	0	< 0.05
	$\searrow$	97	72.9	−0.52	< 0.05

Table 3
Repartition of runners between strong and weak asymmetry in speed time series regardless of positive or negative asymmetry.

Group	Asymmetry	Nb. runner	% Runner
All	Strong	191	70
	Weak	82	30
Men	Strong	162	70.4
	Weak	68	29.6
Women	Strong	29	67.4
	Weak	14	32.6
< 3 h	Strong	91	70
	Weak	39	30
≥ 3 h	Strong	100	69.9
	Weak	43	30.1
Paris	Strong	97	69.3
	Weak	43	30.7
Berlin	Strong	94	70.7
	Weak	39	29.3

An another interesting fact is that we have 12 runners with a Kendall's τ in the range −0.05–0.05. This suggests that a minority of runners can hold a constant speed during a marathon.

To conclude with the trend of the speed time series we clearly observe a majority of negative Kendall's tau values strengthening the idea that spontaneously runners decrease their speed during the race regardless of the gender, performance or race profile. These computations agree with the assessment from the literature [13,14].

3.2. *Speed strong asymmetry behavior*

Let us now study asymmetry behavior of the speed distribution. In order to do so, we use the percentile of the average speed described in Section 2.2.2 by computing the time spent above and below the runners' marathon average speed. Both the whole dataset and strong speed asymmetry categories have been taken into consideration in this section.

First Table 3 presents the repartition of runners between strong and weak asymmetries in speed time series regardless of positivity or negativity. We can see that there is, regardless of gender, performance or race profile, about 70% of runners being strongly asymmetric and 30% being weakly asymmetric.

To go further and study the sign of the asymmetry, Table 4 displays the results for the two categories and the seven groups (All, Men, Women, < 3 h, ≥ 3 h, Paris, Berlin). In most cases, asymmetry is slightly more negative than positive. This means

Table 4
Repartition of runners by whole dataset vs. strong asymmetry and positive vs. negative asymmetry in seed time series.

Group	Asymmetry	Whole dataset		Strong speed asymmetry	
		Nb. runners	% Runners	Nb. runners	% Runners
All	+	119	43.6	75	39.3
	−	154	56.4	116	67.7
Men	+	101	43.9	62	38.3
	−	129	56.1	100	61.7
Women	+	18	41.9	13	44.8
	−	25	58.1	16	55.2
< 3 h	+	71	54.6	43	47.3
	−	59	45.4	48	52.7
≥ 3 h	+	48	33.6	32	32
	−	95	66.4	68	68
Paris	+	54	38.6	33	34
	−	86	61.4	64	66
Berlin	+	65	48.9	42	44.7
	−	68	51.1	52	55.3

Table 5
Subdivided repartition of the runners in the signature for the whole dataset category by the following seven groups: Men, Women, Less than 3 h, Greater than or equals to 3 h, Paris, Berlin, All.

Trend	Asymmetry	Men (%)	Women (%)	< 3 h (%)	≥ 3 h (%)	Paris (%)	Berlin (%)	All (%)
−	−	122 (53.8)	25 (58.1)	56 (43.1)	91 (63.6)	83 (59.3)	64 (48.1)	147 (53.8)
−	+	68 (29.6)	14 (32.6)	51 (39.2)	31 (21.7)	44 (31.4)	38 (28.6)	82 (30)
+	+	33 (14.3)	4 (9.3)	20 (15.4)	17 (11.9)	10 (7.1)	27 (20.3)	37 (13.6)
+	−	7 (3)	0 (0)	3 (2.3)	4 (2.8)	3 (2.1)	4 (3)	7 (2.6)

that runners tend to run more time above their marathon average speed. We can see that for the whole dataset category, 56.4% of the runners have a negative asymmetry. This percentage increase for the strong speed asymmetry category to 67.7% meaning that the weak asymmetry runners tend to have a positive asymmetry.

Only the less than 3 h group for the whole dataset category has a majority of runners with a positive asymmetry. This could be linked to their performance and the fact that they know better how to run a marathon. This idea is accentuated by the percentage of runners of the greater than or equals to 3 h group (66.4%) which is the highest for negative asymmetry.

It seems that the place where the marathons have been run as an importance as well. Indeed, Paris has more negatively asymmetric runners (Whole dataset: 61.4%; Strong speed asymmetry: 66%) than Berlin (Whole dataset: 51.1%; Strong speed asymmetry: 55.3%)

The gender does not seem significant enough to give a conclusion on the tendency of speed asymmetry.

To conclude with the asymmetry behavior of the speed time series, we highlighted the following three facts thanks to Tables 3 and 4:

- 70% of the runners run in a strong asymmetric way regardless of the gender, level of performance and race profile.
- The asymmetry tends to be negative, meaning that runners are spending more time above their marathon average speed than below.
- The asymmetry sign and repartition clearly depends on the level of performance and the race profile but not markedly on the gender.

3.3. *Speed statistical signature*

The study of trend and asymmetry of speed time series taken jointly lead to building up the statistical signature described in Section 2.2.3.

The first significant result appears from Tables 5 and 6 giving the four signature frequencies for the seven groups: Men, Women, less than 3 h, greater-than or equal to 3 h, Paris, Berlin and All. Table 5 displays the results for the whole dataset category while Table 6 shows the results for the strong asymmetry category.

It is clear that the repartition of the four signatures is stable. The frequency is significantly decreasing with respect to the ranking (−−), (−+), (++), (+−) of the signatures.

From Tables 5 and 6, the group "All" is the global statistical signature i.e. repartition of the runners by tendency and asymmetry. In both Tables we can see that the (− −) cluster represents more than half of the runners (whole dataset category: 53.8% ; strong asymmetry category: 59.2%) followed by (− +) and (+ +) representing the rest (whole dataset category: 43.6% ; strong asymmetry category: 39.2%). The (+ −) cluster represents a minority of runners. The main difference

Table 6
Subdivided repartition of the runners in the signature for the strong asymmetry category by the following seven groups: Men, Women, Less than 3 h, Greater than or equals to 3 h, Paris, Berlin, All.

Trend	Asymmetry	Men (%)	Women (%)	< 3 h (%)	≥ 3 h (%)	Paris (%)	Berlin (%)	All (%)
−	−	97 (59.9)	16 (55.2)	47 (51.6)	66 (66)	64 (66)	49 (52.1)	113 (59.2)
−	+	30 (18.5)	10 (34.5)	23 (25.3)	17 (17)	23 (23.7)	17 (18.1)	40 (20.9)
+	+	32 (19.8)	3 (10.3)	20 (22)	15 (15)	10 (10.3)	25 (26.6)	35 (18.3)
+	−	3 (1.9)	0 (0)	1 (1.1)	2 (2)	0 (0)	3 (3.2)	3 (1.6)

between the whole dataset and strong asymmetry categories is the interval between (− +) and (+ +). While (− +) is twice larger than (+ +) in the whole dataset category, they are nearly equals in the strong asymmetry category.

In Table 5 we see that the order of the statistical signature for the whole dataset category is the same regardless of gender, level of performance and race profile (− −) (− +) (+ +) (+ −). In Table 6 we see that the order of the statistical signature for the strong asymmetry category is the same regardless of gender and level of performance (− −) (− +) (+ +) (+ −) but changes for the race profile (− −) (+ +) (− +) (+ −). In both categories, the exact repartition of the runners varies.

4. Discussion

To the best of our knowledge, this is the first study describing the asymmetric behavior of runners during marathons, asymmetry which appears to be independent of any categorization of runners and races. Additionally we introduced a new statistical signature to analyze the trend and the asymmetry jointly. The main findings of this research are as follow:

1. 70% of the runners tend towards having a strong asymmetric behavior in the speed time series while running a marathon regardless of the gender, level of performance and race profile.
2. The asymmetry tends to be negative, meaning that runners are spending more time above their marathon average speed than below.
3. The statistical signature of a marathon keeps the same order (− −) (− +) (+ +) (+ −) whichever the gender, the level of performance and the race profile.

However, we are aware that in this paper, we compared men and women looking at their absolute performance and it would be interesting to study their relative performance compared to world record as well. Beside this reserve regarding the gender effect, thanks to the statistical signature tool we confirmed the fact that marathon runners cannot maintain a target even pace. They are going to reach a point of fatigue which has been described in literature as a matter of high fractional use of VO2max (68 to 100% of VO2max) decreasing during the race [15].

The extreme physical endurance demand has induced a high post-race rectal temperatures due to dehydration and metabolic rate [16,17] introducing a precipitous rise in body temperature from 40.9 to 41.9 °C [18]. The increase in body temperature appears due to the inability of the runner to maintain the well-known "glycogen shunt" necessary for sustaining the muscle rapidity of contraction at the instant of the runner stabilizes. Such rapid contraction requires that the energy is delivered fast so that the muscle has the power requirements of rapid energy expenditure that are ultimately met by a slower averaged consumption of carbon and oxygen from blood [19] as in the last 10 kilometers of a marathon race. Therefore it is the reason why the paradigm of even pace maintained until exhaustion [20,21], leads to a much lower critical speed that using intermittent speed [22]. This impossible recovery of energy among muscle contraction induces an increase of energy cost of running [23] leading the marathoner to suffer a lack of energy while requirements per kilometer increase. In order to reach an optimal performance, runners have to make decisions about how and when to invest their energy. In other words marathoners have to deal with exercise intensity regulation during the race [24]. Although the physiological regulation of pacing strategy during exercise is still in debate for a marathon, runners tend to adopt a constant speed and seem to try keeping it for as long as possible.

Pacing strategy has been rather described as the efficient use of energetic resources during athletic competition so that all available energy stores are "used before finishing a race, but not so far from the end of a race that a meaningful slowdown can occur" [25]. Most of the time, marathoners are unsatisfied with their race. We distinguish two kind of behavior. First, runners considering that they did not gave their best (increasing trend in speed time series). Second, runners being completely exhausted due to the famous "marathon wall" [26].

The present study showed that the non optimal employed pacing strategy in non elite marathon runners was independent of gender, level of performance and marathon profile. This goes into the same direction as a prior study done on the recent world record marathon runners using also a statistical analysis of high frequency (1 km) split data [27]. This last study may conclude that for realistic marathon conditions (i.e. no treadmill), an approximation to an optimal pacing strategy should rest on two principles:

- Principle 1 (the even principle). Where on-course wind and/or gradient variations are insignificant, minimize variations from average power.
- Principle 2 (the parallel principle). Where on-course wind and gradient variations exist, apply small (0%–10%) variations in power (speed) in parallel to environmental variations to maintain a constant running speed (i.e. exert more (less) power uphill (downhill) or into head-winds (tail winds).

This assumption of the ideal race at constant power [28–33] is based on the 1973 Keller's model identifying a runner with a Newtonian particle. However, a prior study has interestingly reported that older runners have a still more even slow pace that the younger one. Here we did not focus on the age effect, but considering that running performance has a structural basis [34] and that muscle mass decreases with age, older runners cannot bear speed variation due to their narrow metabolic scope associated with their loss of strength that is amplified during a marathon which is often described as a "second (slow) death" by the runners at the arrival.

The future of marathon performed with the integrity of human homeostasis is now more and more considered to be associated with the concept of self-paced run rather than having to follow a constant target speed [35–37]. This strategy is adopted in elite runners especially because they have a sufficient metabolic scope [20], a high speed reserve between average marathon and 1000 m paces [38]. As underlined above oscillations in energy metabolism is associated with the recovery of phosphocreatine pattern necessary for maintaining the muscle strength to take off the ground at each step [39]. Therefore, the future of performance while running a marathon will probably be associated with a new paradigm of dissipative system with dynamic instabilities. The visible part is the speed variation that cannot be any more considered as a mistake but rather as an intelligent fit of metabolism adaptation in real time. That is why, here we propose a "qualitative" approach of the marathon race beyond the only recognized performance criteria: the time. The performance, beyond the final time, is also a way to succeed a marathon. A qualitative analysis could be a necessary step to break the two-hour marathon which has been a real recurrent issue for almost 10 years [40,41].

5. Conclusion

Running a marathon at a perfect even pace seems nearly impossible. The research on that matter could explore other pacing strategies and apply them in real conditions in order to have a proper validation. We think that speed variation is the optimal way of running but we still need to find a way to build custom-made strategies by clustering by gender, level of performance or race profile for instance. Meanwhile we can think of two research opportunities to go further regarding this paper. One would be to try to find universal speed patterns for marathons and see if these match the statistical signature. This could be the first step for clustering runners with the idea of predicting pacing strategies in future works. The other one would be to study heart rate trend and asymmetry and then combine the results with those from this paper. The cardiac cost could help us to link both speed and heart rate time series at the same time.

References

[1] Running-USA, 2016 Running USA Annual Marathon Report.
[2] L.A. Léger, R.J. Ferguson, Effect of pacing on oxygen uptake and peak lactate for a mile run, Eur. J. Appl. Physiol. Occup. Physiol. (1974).
[3] M. Ariyoshi, K. Yamaji, R. Shephard, Influence of running pace upon performance: Effects upon treadmill endurance time and oxygen cost, Eur. J. Appl. Physiol. Occup. Physiol. (1979).
[4] V.L. Billat, J. Slawinski, M. Danel, J.P. Koralsztein, Effect of free versus constant pace on performance and oxygen kinetics in running, Med. Sci. Sports Exerc. (2001).
[5] F. Cottin, Y. Papelier, F. Durbin, J.P. Koralsztein, V.L. Billat, Effect of fatigue on spontaneous velocity variations in human middle-distance running: use of short-term fourier transformation, Eur. J. Appl. Physiol. (2002).
[6] L.E. Sandals, D.M. Wood, S.B. Draper, D.V.B. James, Influence of pacing strategy on oxygen uptake during treadmill middle-distance running, Int. J Sports Med. (2006).
[7] M. Garcin, M. Daniel, V.L. Billat, Perceptual responses in free vs. constant pace exercise, Int. J. Sports Med. (2008).
[8] M.R. Ely, D.E. Martin, S.N. Cheuvront, S.J. Montain, Effect of ambient temperature on marathon pacing is dependent on runner ability, Med. Sci. Sports Exerc. (2008).
[9] J.B. Keller, A theory of competitive running, Phys. Today (1973).
[10] M.G. Kendall, A new measure of rank correlation, Biometrika (1938).
[11] A. Agresti, Analysis of Ordinal Categorical Data, second ed., John Wiley and Sons, 2010.
[12] M. Kendall, A. Stuart, The Advance Thory of Statistics, Griffin and Co, 1963.
[13] A. Santos-Lozano, P.S. Collado, C. Foster, A. Lucia, N. Garatachea, Influence of sex and level on marathon pacing strategy. Insights from the New York City Race, Int. J. Sports Med. (2014).
[14] V.L. Billat, L. Mille-Hamard, Y. Meyer, E. Wesfreid, Detection of changes in the fractal scaling of heart rate and speed in a marathon race, Physica A (2009).
[15] M. Maron, S.M. Horvath, J.E. Wilkerson, J.A. Gliner, Oxygen uptake measurements during competitive marathon running, J Appl. Physiol. (1976).
[16] S.N. Cheuvront, E.M. Haymes, Thermoregulation and marathon running, Sports Med. (2001).
[17] E.F. Coyle, Physiological regulation of marathon performance, Sports Med. (2007).
[18] M.B. Maron, J.A. Wagner, S.M. Horvarth, Thermoregulatoryresponses during competitive marathon running, J. Appl. Physiol. (1977).
[19] The "glycogen shunt" in exercising muscle: A role for glycogen in muscle energetics and fatigue, 2001.
[20] C.C. Peterson, K.A. Nagy, J. Diamond, Sustained metabolic scope, Proc. Natl. Acad. Sci. USA (1990).
[21] A. Vanhatalo, A.M. Jones, M. Burnley, Application of critical power in sport, Int. J. Sports Physiol. Perform. (2011).
[22] R.H. Morton, V.L. Billat, The critical power model for intermittent exercise, Eur. J. Appl. Physiol. (2004).
[23] C. Hausswirth, D. Lehenaff, Physiological demands of running during long distance runs and triathlons, Sports Med. (2001).
[24] B.L.M. Smits, G.-J. Pepping, F.J. Hettinga, Pacing and decision making in sport and exercise: the roles of perception and action in the regulation of exercise intensity, Sports Med. (2014).
[25] R. Tucker, T. Noakes, The physiological regulation of pacing strategy during exercise: a critical review, Br. J. Sports Med. (2009).
[26] J.J. Diaz, E.J. Fernández-Ozcorta, J. Santos-Concejero, The influence of pacing strategy on marathon world records, Eur. J. Sport Sci. (2018).
[27] S.D. Angus, Did recent world record marathon runners employ optimal pacing strategies? J. Sports Sci. (2014).
[28] T.A. Haney, J. Mercer, A description of variability of pacing in marathon distance running, Int. J. Exercise Sci. (2011).
[29] R. Maronski, K. Rogowski, Minimum-time running: A numerical approach, Acta Bioeng. Biomech. (2011).

[30] W. Woodside, The optimal strategy for running a race (A mathematical model for world records from 50 m to 275 km), Math. Comput. Modell.: Int. J. (1991).
[31] M.A. Townsend, Road-racing strategies, Med. Sci. Sports Exerc. (1982).
[32] W.G. Pritchard, Mathematical models of running, Soc. Ind. Appl. Math. (1993).
[33] A.B. Pitcher, Optimal strategies for a two-runner model of middle-distance running, SIAM J. Appl. Math. (2009).
[34] P.G. Weyand, J.A. Davis, Running performance has a structural basis, J. Exp. Biol. (2005).
[35] D. Van-Biesen, F. Hettinga, K. McCulloch, Y.C. Vanlandewijck, Pacing profiles in competitive track races: regulation of exercise intensity is related to cognitive ability, Front. Physiol. (2016).
[36] J.S. Hogg, J.G. Hopker, A.R. Mauger, The self-paced VO2max test to assess maximal oxygen uptake in highly trained runners, Int. J. Sports Physiol. Perform. (2017).
[37] A.E. Iyoho, L.N. MacFadden, L.J. Ng, Prediction of performance reduction in self-paced exercise as modulated by the rating of perceived exertion, Eur. J. Appl. Physiol. (2015).
[38] V.L. Billat, A. Demarle, J. Slawinski, M. Paiva, J.P. Koralsztein, Physical and training characteristics of top-class marathon runners, Med. Sci. Sports Exerc. (2001).
[39] S. Iotti, M. Borsari, D. Bendahan, Oscillations in energy metabolism, Biochim. Biophys. Acta (2010).
[40] M.J. Joyner, J.R.R.A. Lucia, The two-hour marathon: who and when? J. Appl. Physiol. (2011).
[41] R. Tucker, J. Santos-Concejero, The unlikeliness of an imminent sub-2-hour marathon: historical trends of the gender gap in running events, Int. J. Sports Physiol. Perform. (2017).